Motivating Modal Semantics

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1 Where Semantics Comes From

- Motivating mysteries:

  Consequence Some claims 'follow' from others and some claims don't 'follow' from others. Why?

  - (3) seems to follow from (1) and (2):

    (1) If Drake is drinking, he is singing

    (2) If Drake is singing, he is offending someone

    (3) If Drake is drinking, he is offending someone

  - (5) does *not* seem to follow from (4):

    (4) It is not true that if God exists, he is made of spagetti

    (5) God exists

  Consistency Some claims are compatible, others are not. Why?

  - E.g. (6) and (7)

    (6) Drake is drinking

    (7) Drake is not drinking

- Classical logic provides two sets of tools for addressing these mysteries:
  - Semantics (model theory)
  - Proof theory
- Let's compare what these two methods say about Consequence
- First, what's a 'claim'?
  - A natural language sentence used by a person to communicate something
- Both start with a common assumption:
  - The natural language sentences we use to make claims can be translated into sentences of a formal language
    - ► Such as First-Order Logic

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- E.g. (1) translates as the formula $D \rightarrow S$
- Both semantic and proof-theoretic approaches offer an account of what it is for one of these formulas to be a consequence of others
  - Model Theory: $P_1, \ldots, P_n \vDash C$
  - Proof Theory: $P_1, \ldots, P_n \vdash C$
- So their explanations of why some claims follow from others is based on the linguistic signals used, not *how* they are used
  - Though the sentences may inherit their meaning from their use
  - It is because they have that meaning that some claims follow from others
  - Not because of the details of how those sentences are used in particular contexts
- Proof Theoretic explanation of why (3) follows from (1) and (2):
  - Each connective is associated with two 'primitively compelling' rules of inference

Conditional Introduction (→ Intro)

	│ P
	│ ⋮
	│ Q
▷	$P \rightarrow Q$

Conditional Elimination (→ Elim)

	$P \rightarrow Q$
	⋮
	P
	⋮
▷	Q

> **Proof Theoretic Consequence ($\vdash$)** Consequences are those formulas which can be derived using the primitive rules

  - Translate:
    - ► (1) is $D \rightarrow S$
    - ► (2) is $S \rightarrow O$
    - ► (3) is $D \rightarrow O$
  - Prove:

1	$D \rightarrow S$	
2	$S \rightarrow O$	
3	│ D	
4	│ S	→ **Elim**: 1, 3
5	│ O	→ **Elim**: 2, 5
6	$D \rightarrow O$	→ **Intro**: 3-5

- Semantic explanation of why (3) follows from (1) and (2):
  - Each atomic sentence is either true – 1 – or false – 0.
    - Call an assignment of truth-values to every atomic sentence a **valuation** V
      - E.g. $V(\mathsf{D}) = 1, V(\mathsf{S}) = 1, \ldots$
    - V records all of the atomic facts about the world
  - Each connective is associated with a truth-function

Material Conditional

P	Q	P → Q
1	1	1
1	0	0
0	1	1
0	0	1

  - So V suffices to give a truth-value to every formula
    - So it doesn't just record all of the atomic facts about the world, it effectively records all of them!

Semantic Consequence (⊨) $\mathsf{P}_1, \ldots, \mathsf{P}_n \vDash \mathsf{C}$ iff there is no valuation that makes $\mathsf{P}_1, \ldots, \mathsf{P}_n$ true and C false

    - Intuition: it's *impossible* for the premises to be true while the conclusion is false
    - No way the world could be could make the premises true and the conclusion false
  - Every V that makes $\mathsf{D} \to \mathsf{S}$ and $\mathsf{O} \to \mathsf{S}$ true, also makes $\mathsf{D} \to \mathsf{O}$ true:

D	S	O	D → S	S → O	D → O
1	1	1	1	1	1
1	1	0	1	0	0
1	0	1	0	1	1
1	0	0	0	1	0
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	0	1	1
0	0	0	0	1	0

  - Invalidity (lack of consequence) is easy to show:

    - Find a valuation where the premises are all true and the conclusion false
- Many regard semantic definition of consequence as basic, but why? (Tarski 1956)
  - Others regard proof-theoretic consequence as basic (Dummett 1991)
  - Philosophy of logic vs. natural language semantics
    - Normative vs. descriptive theory of consequence
- What is the mathematical relationship between ⊢ and ⊨?
  - Soundness & Completeness
    - Soundness: if ⊢ then ⊨
    - Completeness: if ⊨ then ⊢
    - FOL is sound and complete
      - FOL: $=, \neg, \vee, \wedge, \to, \leftrightarrow, \forall, \exists$, plus names and predicates
    - But FOL plus arithmetic is not
      - Does this mean ⊨ is better?
  - Undecidability of invalidity in FOL
    - Whether or not a given proof is valid is decidable
      - With enough resources and time, you are guaranteed to get an answer
    - But whether or not a proof exists is not
      - Not guaranteed to get an answer as to whether an argument is valid
    - This is at least uncomfortable for proof theoretic advocates
      - For them, the facts about positive consequence are constituted by the existence of the proof
      - But negative facts about consequence cannot be constituted that way!
- Explanatory priority?
  - To many, semantic explanations of consequence seem better somehow, but how?
  - We want to answer the question of why certain inferences are good,
    - Proof theoretic answer says: because these more basic inferences are good
    - So it is not a reductive answer: why are those more basic inferences good
  - The semantic answer seems more reductive:
    - Consequence is reduced to truth-preservation
    - Since it is independently attractive to assume that sentences have truth-values, it looks like the semantic theory is more parsimonious
    - But is it really reductive?

- ▷ Don't we assume certain inferences are primitively compelling when stating the semantics? (Carroll 1895; Quine 1936)
    - ▷ For a correct theory of inference, don't we need to assume that some valuations are incompatible, e.g. *x is blue* and *x is red*? (Wittgenstein 1922: 6.3751)
  - ▶ Maybe its appeal lies in that it unifies two questions: truth and consequence

- Generality?
  - ◦ We all recognize that (3) is a consequence of (1) and (2)
    - ▶ Even assuming it's because we can all subconsciously construct a proof, surely we don't all construct exactly the same proof
    - ▶ Yet there is some sense in which we are doing the 'same thing'
  - ◦ The semantic approach abstracts away from calculation entirely
    - ▶ It allows us to capture which consequences we recognize without saying how we recognize them
    - ▶ This is the benefit of characterizing inference in 'worldly' terms
  - ◦ Reminiscent of Marr's (1982) distinction of mathematical vs. representational level
    - ▶ And Newell's (1981) *knowledge level*

2 Why Modal Semantics?

2.1 Modals

- Consider the following intuitive entailment:

(8) a. Obama isn't necessarily going to win the election

b. It's possible that Obama will loose the election

- It seems that if (8a) is true, (8b) has to be true as well
- This inference relies crucially on the modal adverbs *necessarily* & *possible*
- Perhaps we just need to add modal operators to propositional logic?
  - ◦ Let $\Box\mathsf{A}$ mean *It is necessarily true that A*
  - ◦ Let $\Diamond\mathsf{A}$ mean *It is possible that A*
- Okay, but can we say what these operators mean in terms of truth-functions?
  - ◦ How do we complete this table?

Win	◇ Win
1	1
0	?

  - ◦ Intuitively, just knowing that Obama didn't win does tell us enough to know whether it was possible for Obama to win
    - ▶ So neither choice is intuitively appropriate
  - ◦ Both choices get things wrong:
    - ▶ It's consistent to say that *Obama didn't win, but it was possible for him to win*
      - ▷ So 0 is wrong
    - ▶ Also consistent: *Obama didn't win, and it wasn't even possible for him to win*
      - ▷ So 1 is wrong too!
- The truth-value of W doesn't determine the truth-value of $\Diamond\mathsf{W}$
- Parallel difficulties apply to $\Box$
- Intuitively, what's going wrong in this semantics?

2.2 Conditionals

- We noted that (4) does **not** follow from (5), what does the classical semantics predict?
  - ◦ If $\neg(\mathsf{G} \to \mathsf{S})$ is true then $\mathsf{G} \to \mathsf{S}$ is false, in which case G is true!
    - ▶ The classical semantics predicts that this inference should be good!

Material Negation $\neg(\mathsf{P} \to \mathsf{Q}) \vDash \mathsf{P}$

- The following are also intuitively bad, but materially valid inferences:

(9) Bob didn't dance, so if Bob danced, he's a turnip.

(10) If it rains, it won't pour. So, if it pours it won't rain. (Adams 1975: 15)

Antecedent Negation $\neg\mathsf{P} \vDash \mathsf{P} \to \mathsf{Q}$

Contraposition $\mathsf{P} \to \mathsf{Q} \vDash \neg\mathsf{Q} \to \neg\mathsf{P}$

- Maybe we picked the wrong truth-function for $\to$?
  - ◦ It is the only option that renders $\to$ **Intro** and $\to$ **Elim** valid!
- One option: pursue a different semantics
- Another option: our judgements are really about *claims made by people*
  - ◦ A claim not only involves a sentence with a meaning, but a speaker who is attempting to get something across
- Grice (1989a) pursues the second option
  - ◦ Classic criticism: Jackson (1979)

Conversational Principle Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1975: 26)

Quality Maxim (i) Make your contribution as informative as is required. (ii) Do not make your contribution more informative than is required. (Grice 1975: 26)

Manner Maxim (i) Avoid obscurity of expression. (ii) Avoid ambiguity. (iii) Be brief (avoid unnecessary prolixity). (iv) Be orderly.

- Antecedent negation: after you say $\neg P$, $P \to Q$ becomes unassertable
  - Once P is taken to be false, $P \to Q$ cannot serve to communicate anything that $\neg P$ did not communicate, even though $P \to Q$ is true!
    - We confuse this inappropriateness for falsity
    - Much like when I say *I have three kids* when I have four
  - This seems to violate both the Quality and Manner Maxims
  - Does this explanation overgeneralize? Should modus ponens sound weird?
- Material Negation:
  - When you say *It is not true that if God exists, he is made of spagetti*, you are not expressing the negation of a conditional, but rather a refusal to assert the conditional.
    - You should refuse to assert it because there is a better way of expressing yourself: *If God exists, he is not made of spagetti.*
  - This response seems unprincipled:
    - Why does negation get to mean refusal to accept sometimes and a truth-function at other times?
- Contraposition:
  - Grice didn't make much headway on this one

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