Indicative Conditionals, Strictly

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A truth-functional analysis of conditionals is now widely held to be so untenable even Gricean heroics cannot save it. But a similar consensus about the prospects of a strict conditional analysis has recently broken into discord, led by the voices of Warmbrōd (1983), Veltman (1985, 1986) and Gillies (2004, 2009, 2010). While some merits of the resurgent strict analysis have been enumerated, no general comparison of it to other analyses (i.e. variably-strict and probabilistic ones) has been undertaken. Indeed, many central questions about the truth-conditions, logic and pragmatics of strict analyses remain unsettled and even unasked. This paper focuses on strict analyses of *indicative* conditionals and attempts to fill these gaps.[1] A preferred articulation of the strict analysis emerges and its numerous advantages over variably-strict and probabilistic analyses are more thoroughly detailed. A new class of counterexamples to a suite of patterns typically validated by strict analyses is presented and diagnosed. They figure prominently in this comparison.

1 Introduction

If Peirce is to be trusted, a strict analysis of conditionals was first developed by Philo the Logician, a member of the early Hellenistic Dialectical School.[2] But Peirce himself seems to have been the first modern strict conditional theorist.

> ...[*P*]*ossibility* may be understood in many senses; but they may all be embraced under the definition that that is possible which, in a certain state of information, is not known to be false. By varying the supposed state of information all the varieties of possibility are obtained. Thus, *essential* possibility is that which supposes nothing to be known except logical rules. *Substantive* possibility, on the other hand, supposes a state of omniscience.

[1] This focus on indicative conditionals is a mere convenience since the strict analysis that emerges could be integrated with the a unified analysis of conditionals offered by Starr (forthcoming).

[2] Bobzien (2011: §3.1) presents Philo as a material implication theorist. But according to Peirce (1896: 33) the Philonian view is more nuanced. It is only because of the additional commitment that logic concern *substantive* possibilities, and thus limit itself to one possibility, the actual world, that Philonians embrace the logic of material implication. Then again, Peirce may be reading his modal distinctions back into Philo. I first learned about Peirce's views from Copeland (2002) and Zeman (1997).

> ...[A]n ordinary Philonian conditional is expressed by saying, 'In any possible state of things, i, either A_i is not true, or B_i is true.' (Peirce 1896: 32-3)
> ...[T]he consequent of a conditional proposition asserts what is true, not throughout the whole universe of possibilities considered, but in a subordinate universe marked off by the antecedent.
> (Peirce in the *Grand Logic* [1893-4]; Hartshorne & Weiss 1933: 4.435)

This early version of the view is not only striking in its earliness. It is striking in two more important ways: its appeal to *states of information* and its ambivalence between two articulations of the view. These two articulations actually demarcate an important choice-point in developing a strict analysis that will be confronted below:

- Conditionals universally quantify over possibilities, saying that the material conditional holds at each possibility.
- Conditionals assert the consequent against a space of possibilities restricted by the antecedent.

The ambivalence here is between an account where strict conditionals describe a relation between possibilities and one where they are characterized in terms of a dynamic process involving the space of possibilities. Peirce's proposal that this space of possibilities be defined in terms of the information available to the agents' will be a crucial component of a workable strict analysis. Surprisingly, this component turns out to be in tension with the descriptive, but not the dynamic, view of strict conditionals (§2). The two accounts are, given a particular assumption, formally equivalent (Gillies 2009, 2010). But what has not been observed in the literature is that this assumption is incompatible with treating the space of possibilities as a state of information.

One appeal of treating (spaces of) possibilities as information states comes in developing an adequate logic of conditionals. Pointing to the many failures of the material conditional analysis, C.I. Lewis (1914) articulated a new logic for strict conditionals free from those particular failures. However, it was noted by Lewis himself that this view had several similar failures, the so-called paradoxes of strict implication. While Lewis (1914) attempted to justify these results, most remained skeptical (e.g. Strawson 1948). The skepticism was bolstered by Stalnaker (1968, 1975) and Adams (1965, 1975). They offered counterexamples to seemingly attractive patterns of inference (antecedent strengthening, simplification of disjunctive antecedents, transitivity and contraposition) validated by material and strict conditionals alike. They also offered much weaker conditional logics that validated none of these patterns. But there was still hope for the strict conditional. Working independently, Warmbrōd (1983) and Veltman (1985, 1986) made a startling discovery. If the space of possibilities is construed as an agent's state of information, *all* of the apparent counterexamples to C.I. Lewis' logic required shifts in that body of information which violated a plausible pragmatic constraint. This constraint is in fact one that Stalnaker (1975) and Adams (1975) also endorse: asserting an indicative conditional is felicitous only if its antecedent is possible with respect to agents' information. Thus, it could be claimed that the patterns sound bad not because they are invalid, but because they are conversationally infelicitous. Gillies (2009) introduced another theoretical option: to treat Warmbrōd and Veltman's pragmatic constraints as a semantic presupposition and pair it with a non-classical (dynamic) account of logical consequence where information states shift as premises are accepted. But Gillies' work leaves open two questions that are important choice points in developing a strict semantics. Should these constraints be treated pragmatically or semantically? If treated semantically, how should cases involving failed presuppositions count towards the validity of an inference pattern? As I will argue in §3.1, embedded conditionals provide powerful reasons for treating the constraints semantically. I will then argue that a dynamic Strawsonian definition of validity is needed, where failures of presupposition do not count toward the validity of an inference pattern at all.[3] Initially, there are a few minor surprises in the resulting analysis, but it proceeds broadly in the way suggested by strict conditional theorists. However, a very significant surprise is in store. I will not only present new counterexamples to antecedent strengthening, simplification of disjunctive antecedents, transitivity and contraposition which cannot be explained away as Warmbrōd, Veltman and Gillies propose. But I will show that the dynamic semantics offered actually predicts these patterns to be invalid and articulates exactly why. This allows me to prove that limited instances of these patterns are valid. I argue that the resultant logic compares favorably to Stalnaker (1968, 1975) and Adams (1965, 1975).

A final choice-point arises for those, like myself, that choose the dynamic, procedural version of the strict analysis. This kind of analysis only assigns sentences acceptance conditions, rather than truth conditions (despite doing so in a thoroughly compositional way). But what then are the truth-conditions of indicative conditionals? Are we just not allowed to ask that question, or does it simply become irrelevant? Veltman (1985, 1986) and Gillies (2009: 338) appear to identify the truth conditions of a dynamic conditional with its acceptance conditions. I argue that this is dissatisfying, and introduce an alternative inspired by Peirce and his influences' distinction between essential and substantive possibilities. The truth conditions of a dynamic conditional in a world w are given by its acceptance conditions in a state of omniscient information about w. The dynamic strict semantics, with the presupposition that its antecedent be possible, then predicts three-valued truth conditions for indicative conditionals: they are true when antecedent and consequent are true, false when antecedent true and consequent false, and undefined otherwise. This too is a view with a long history and many adherents (e.g. de Finetti 1936: 35; Jeffrey 1963; Belnap 1973; McDermott 1996; Milne 1997). Its chief virtue is that it can can solve a puzzle introduced by Lewis (1975) concerning the interaction of conditionals with existential, universal and proportional adverbs of quantification, e.g. *might*, *must* and *probably* (Huitink 2008).[4] Its chief vice, elaborated below in §4.2, is that when articulated in a truth-conditional semantics it generates a very implausible logic. I show that the dynamic strict conditional logic outlined here does not suffer from this vice. Even though the trivalent truth conditions are not the *meaning* of a conditional on the dynamic strict theory, they can be made available to operators such as *probably* when needed. I show that this allows a semantics of *probably* which meets the Lewis-Kratzer demands (§4.1). Since Gillies (2010) has already shown that a strict dynamic theory can meet these demands with respect to universal (*must*) and existential (*might*) adverbs, the resulting theory meets the Lewis-Kratzer challenge more completely.

[3] The term and concept of *Strawsonian entailment* was introduced by von Fintel (1999), drawing on Strawson (1952: 173-9).

[4] To some, it is more than a mere puzzle since it proposes to show that *no conditional connective whatsoever* could explain Lewis' pattern.

2 How to be Strict: descriptive or dynamic?

If given the choice between a 'descriptive' and a 'dynamic' theory, who – other than your accountant – would pick the 'descriptive' one? Marketing gimmicks aside, I think there are real reasons for preferring a dynamic strict conditional analysis to a descriptive one. But this emerges only once the two are formulated precisely.

The ('ordinary Philonian') strict conditional formulated by Peirce (1896: 32-3) *describes* a particular relation holding between antecedent-worlds and consequent-worlds, namely inclusion. This idea can be easily articulated in possible worlds semantics. Every formula is assigned a set of possible worlds (a proposition), the set of worlds it is true in. Disjunction is union, conjunction is intersection and negation is set difference on propositions. In addition to a domain of worlds W, this semantics assumes there is a way of delimiting which worlds are possible with respect to others. This is commonly done with an accessibility relation $R(w, w')$ meaning that w' is possible with respect to w, but I'll opt for the more intuitive and equivalent method of using a function $R(w)$ which returns the set of worlds that are possible with respect to w. Peirce's descriptive strict conditional can then be formulated as in Definition 1.

Definition 1 (Descriptive Strict Conditional v.1)
$[\![\phi \rightarrow \psi]\!]_R = \{w \mid R(w) \cap [\![\phi]\!]_R \subseteq [\![\psi]\!]_R\}$

- All the possible ϕ-worlds are ψ-worlds
- $R(w)$ is the set of worlds possible with respect to w

On this view, a strict conditional describes what's possible with respect to w.[5] This semantics, of course, only crystalized after C.I. Lewis' (1914) purely proof-theoretic studies of strict conditionals and after Kripke's (1963) use of accessibility relations in the semantics of modal logic. As with any modal operator, the question becomes which constraints on R correspond to which logics. This general question will not be of concern here, but one particular constraint needs to be highlighted. Obviously, modus ponens is a nice inference pattern to validate. However, with no constraints the strict conditional in Definition 1 does not. Suppose we are evaluating $\phi \rightarrow \psi$ at w, and w happens to be the lone $\phi \wedge \neg\psi$-world. Without assuming that $w \in R(w)$ it could very well be the case that all the ϕ-worlds in $R(w)$ are ψ-worlds, e.g. $R(w) = W - \{w\}$. But then the semantics predicts that $\phi \rightarrow \psi$ and ϕ are true at w, even though ψ is false at w. So, unless every world is possible with respect to itself, modus ponens fails.

Reflexivity For all $w\colon w \in R(w)$

This is the sole constraint on R that will be relevant below.

While the basic strict semantics of Definition 1 has its merits, Gillies (2004, 2009, 2010) has shown that a slightly modified version solves a wide range of puzzles and is equivalent to a more dynamic version of the strict semantics.[6] One key part of this

[5]Peirce actually said that a strict conditional describes the material conditional holding at all worlds, where $[\![\phi \supset \psi]\!] = (W - [\![\phi]\!]_R) \cup [\![\psi]\!]_R$. But, the two formulations are equivalent: $R(w) \subseteq ((W - [\![\phi]\!]_R) \cup [\![\psi]\!]_R)$ holds if and only if $(R(w) \cap [\![\phi]\!]_R) \subseteq [\![\psi]\!]_R$.

[6]This 'context shifting' component is found in others work as well, e.g. Kratzer (1991: 648, Def.13), Yalcin (2007: 998) and Charlow (2013), and is arguably an immediate consequence of thinking about conditionals along the lines of the Ramsey Test and interpreting the C parameter epistemically.

version is refining the way R, and its associated space of possibilities, is conceptualized. As Gillies (2009: 329) proposes: "A context determines the set of possibilities compatible with the relevant information in that context." To signal this change I will switch to writing $C(w)$ rather than $R(w)$. Intuitively, the idea is that a context C, whatever it is, determines sets of possibilities $C(w), \ldots, C(w')$ each of which is compatible with the information that is relevant in C. On this model, information is just a set of worlds: it says we are in some of these worlds and none of those. So there should be some set of worlds i_C that is the relevant information in C. More formally, the idea comes to this: $C(w)$ will be some subset of i_C, for every w. This change in how the space of possibilities is interpreted is crucial. It motivates a small but key change in the semantics. In Definition 1, the consequent is interpreted against the same R that the whole conditional is. But, once one switches to C there is something to the idea that the consequent should be interpreted with respect to a *shifted* C. After all, there is something to Ramsey's (1931: 247) idea that interpreting an indicative conditional involves hypothetically adding the antecedent to one's stock of information and assessing the consequent on that basis. Indeed, this also seems required to take Peirce's proposal literally that the consequent of a conditional asserts what is true, not throughout the whole universe of possibilities, but in a subordinate universe marked off by the antecedent. Definition 2 achieves this, where C is shifted to C_ϕ in the consequent. ($C_\phi(w)$ returns the ϕ-worlds which $C(w)$ would have returned).

Definition 2 (Descriptive Strict Conditional v.2)
$[\![\phi \to \psi]\!]_C = \{w \mid C(w) \cap [\![\phi]\!]_C \subseteq [\![\psi]\!]_{C_\phi}\}$

- All the contextually-live ϕ-worlds are ψ-worlds
- $C(w)$ is the set of live worlds with respect to w
- $C_\phi(w) = C(w) \cap [\![\phi]\!]_C$, for all w

This might seem like a small, fiddly difference whose only virtue is paying homage to Peirce and Ramsey. But this difference radically changes the way modals and conditionals are interpreted when they are embedded in the consequent. Gillies (2004) shows that this allows one to see the fallacy involved in McGee's (1985) counterexample to modus ponens. Gillies (2009) then shows that it can diffuse an argument advanced by Gibbard (1981) and Edgington (1995) which proposes to show that no plausible propositional analysis of indicatives stronger than the material conditional is possible. Finally, Gillies (2010) shows that it also holds the key for correctly predicting two equivalences involving modals; equivalences which had been argued to be impossible for any connective-based analysis of conditionals to predict (Lewis 1975; Kratzer 1986).[7] This impressive assault on received wisdom provides benefits well-worth preserving by any strict analysis. But, there's a hitch concerning Reflexivity.

Just as with the basic strict analysis, the refined analysis in Definition 2 must assume Reflexivity to uphold modus ponens. It is no surprise, then, that Gillies (2009: 329) explicitly requires this. For his purposes, it is an innocuous assumption. But in offering a general picture of strict conditionals it must be squared with the interpretation of C as determining the set of possibilities compatible with the relevant information

[7] Basically: the equivalence between *If A then must B*, *If A then B* and *Must, if A then B* and that between *If A then might B* and *Might, A and B*.

in that context. Set aside the question of just which information is relevant in contexts – the speaker's, the hearer's, the mutual presuppositions, etc. – and just focus on the fact that this is (a) information and (b) some agents' information. If it's just information, then it is simply a set of worlds – what I called i_C above – not a function from worlds to sets of worlds. If it belongs to agents, it is sometimes wrong, or it is at least logically possible for it to be wrong. After all, in most, if not all, conversations the information taken for granted by any given agent present, or peripherally implicated in, the conversation will include something false.[8]

Perhaps the right reply is that C shouldn't be interpreted as determining sets of possibilities compatible with contextual information. But surely it can't be left *uninterpreted*. And some interpretations simply don't make sense. For example, consider the view on which the modal facts of w, i.e. what's possible with respect to w, is a fact about w's internal constitution and dispositions. On this view, those facts *change* depending on whether you are looking at them from a stand-alone sentence or the consequent of a conditional. Indeed, any view on which C is not fixed by the perspective of an agent is going to be incoherent. It makes sense that our perspective on the possibilities will vary as we evaluate a sentence. But in what good sense do the possibilities themselves actually vary? Isn't that just idealism about modal space? So it seems that if we want the shifty semantics, and we need an informational, perspectival interpretation of C. But in that case, we cannot in good faith require reflexivity.

The above argument can by bolstered by another, offered in a slightly different form by Starr (forthcoming). As Stalnaker (1975) proposed, the basic semantic distinctions between indicative and subjunctive conditionals come to the fact that indicative antecedents concern possibilities that are 'live' in the context (the *contextual possibilities*) while subjunctive antecedents may reach beyond those possibilities. This proposal explains a lot of data. For example, one can felicitously assert the subjunctive (1) after denying the antecedent, i.e. after removing antecedent-worlds from the contextual possibilities. The same is not true for the indicative (2).

(1) Paula Radcliffe didn't run on Tuesday. If Paula Radcliffe had ran on Tuesday, everyone else would have lost.

(2) Paula Radcliffe didn't run on Tuesday. #If Paula Radcliffe ran on Tuesday, everyone else lost.

Now, suppose that the strict semantics is in place, and we are evaluating an indicative conditional in the actual world $w_@$. Then the antecedent-worlds amount to $C(w_@) \cap [\![\phi]\!]_C$. As long as C is reflexive, $C(w_@)$ cannot be the contextual possibilities. After all, the actual world is virtually always eliminated from the contextual possibilities through false belief, negligence and general mischief. So the actual world $w_@$ can't be in $C(w_@)$. Thus, we are forced to choose between C being reflexive and our best explanation of how indicatives and subjunctives differ. That choice is decidedly grim given that modus ponens relies on reflexivity.

Fortunately, there is another way of formulating a strict theory which preserves Gillies' results and modus ponens while forgoing reflexivity. It instead gives up the

[8]It is doubly unhelpful to require that the information be veridical (perhaps, knowledge). First, that prevents applying the theory to everyday conversation. Second, knowledge only assures that $w_@ \in C(w_@)$, in which case modus ponens will still fail because of some non-actual world w such that $w \notin C(w)$.

descriptive element of the previous proposals by employing a version of dynamic semantics called *update semantics* (Veltman 1996). This dynamic analysis has already been presented in Gillies (2004, 2009, 2010), but it is argued to be equivalent to the semantics of Definition 2 (Gillies 2009: §6). What that argument does not highlight is that the dynamic semantics does not need to assume anything like Reflexivity to validate modus ponens. As I have shown above, that is a rather important difference.

What does it mean to give up the descriptive element of the strict semantics, and how could that be relevant at all to getting by without Reflexivity? Let me build up to answering that. The descriptive theory embraces a limited form of dynamics by having a parameter that shifts in a way that captures agents' informational perspective. But, at the end of the day, the semantics still describes some relation among possibilities from a global perspective on those possibilities. A more thoroughly dynamic account would instead just say how accepting a conditional transports us from one informational perspective to another. Instead of thinking about the meaning of a sentence as a proposition – as referring to some region of logical space where a relation holds – think of it as a recipe for locating oneself in logical space, i.e. transitioning from one state of information s (a set of worlds) to another s'. In case that sounds too poetic, here's the math behind the koan. The meaning of a sentence ϕ is a function $[\phi]$ which maps s to s'. I write $s[\phi] = s'$ to mean that the result of applying ϕ to s is s', or as I will say s' is the result of updating s with ϕ. While some updates, like A, will simply eliminate worlds, the $\neg\mathsf{A}$-worlds, conditionals perform a different kind of update called a *test*. On the dynamic analysis, a strict conditional tests that after hypothetically accepting the antecedent in s, only consequent-worlds remain, i.e. $s[\phi][\psi] = s[\phi]$. If this test is passed, s remains the same; otherwise failure $\varnothing$ results (more on this shortly).

Definition 3 (Dynamic Strict Conditional)

$$s[\phi \rightarrow \psi] = \begin{cases} s & \text{if } s[\phi][\psi] = s[\phi] \\ \varnothing & \text{otherwise} \end{cases}$$

- $\phi \rightarrow \psi$ tests that all ϕ-worlds in s are ψ-worlds

Now, suppose you have an s which passes the test imposed by $\phi \rightarrow \psi$. If you gain the information that ϕ, ψ will be supported by the resulting information state, since the conditional guaranteed that all ϕ-worlds in s are ψ-worlds. Thus, modus ponens is upheld without any assumptions about reflexivity. Indeed, the semantics traffics only in information states and operations on them.[9] It does not provide immediate means for evaluating how an utterance of a conditional matches up with the world. In doing so, it remains within the perspective of an agent in a state of information. Reflexivity is not only unnecessary, it's inapplicable.

But it is worth pausing. When I said modus ponens was upheld, I did not say that anytime ϕ and $\phi \rightarrow \psi$ are true, then ψ is true. I said that anytime you update with ϕ and $\phi \rightarrow \psi$ you end up in a state which *supports* ψ. The main logical concept in a dynamic semantics is not truth, but rather *support*. Support tracks which sentences your information entitles you to accept.

[9] Yalcin (2007: 998) offers a semantics that achieves this same effect within a truth-conditional, yet still non-descriptive, semantics:

$$[\![\phi \rightarrow \psi]\!]_{w,s} = 1 \iff \forall w' \in s_\phi : [\![\psi]\!]_{w',s_\phi} = 1$$

where s_ϕ is the maximal non-empty subset of ϕ-worlds in s. The dynamic one generates very different truth-conditions which are essential for the applications discussed in §4.

Definition 4 (Support) $s \vDash \phi \iff s[\phi] = s$

A conclusion follows from some premises just in case accepting the premises in any state of information leads to one where the conclusion is supported.

Definition 5 (Dynamic Consequence)
$\phi_1, \ldots, \phi_n \vDash \psi \iff \forall s\colon s[\phi_1]\cdots[\phi_n] \vDash \psi$

- Updating any s with the premises produces a state that supports the conclusion.

The classical conceptions of truth and consequence are in fact special cases of support and dynamic consequence.

Definition 6 (Truth) $w \vDash \phi \iff \{w\}[\phi] = \{w\}$

Definition 7 (Classical Consequence)
$\phi_1, \ldots, \phi_n \vDash_{Cl} \psi \iff \forall w\colon \{w\}[\phi_1]\cdots[\phi_n] \vDash \psi$

So all good inference patterns preserve truth after all.[10] The subject matter of logic has thus not be shifted, but broadened by using a more general notion. Abstractly put, logic is the study of information flow. Classical logic is the sub-branch that studies the flow of perfect information. There is much more to be said about the truth-conditions of indicative conditionals given by Definitions 3 and 6, but that is the topic of §4.

I will now turn to the question of how indicative conditionals can be informative, given the test semantics in Definition 3. If a conditional simply tests s, it is hard to see how it can be informative. After all, it either results in exactly the same state of information as before, or it is a little *too* informative and leads to $\varnothing$. The answer resides in acknowledging that even in a dynamic semantics, one cannot expect the semantics alone to say how a sentence is used to communicate something. Though the pragmatic story I am about to tell can be told in many ways, I'll tell it in my preferred way. What's distinctive about my preferred way of telling the story is that utterance interpretation is viewed as an inference to the best explanation of the utterance, i.e. *abduction*.[11] On the simple picture painted above, communication involves making certain information mutual. In keeping with this, let s be the set of worlds compatible with what the agents are mutually supposing for the purposes of their exchange (Stalnaker 1978). The best explanation of an utterance will then consist in part in specifying the minimal change to this shared information that makes sense of the utterance. Now, suppose the previously shared information s leaves open (contains) $\phi \wedge \neg\psi$-worlds. If someone says $\phi \to \psi$, then the semantic effect of that utterance will be to reduce s to $\varnothing$. But, contradictory utterances don't make sense, so the agents must find the minimal adjustment of s that does make sense of the utterance. Simple: eliminate the $\phi \wedge \neg\psi$-worlds.

This way of understanding the pragmatics of conditionals has a highly desirable feature. It allows to the agents' to coordinate s with a global feature of some agent's private state without enjoining the numerous disagreements there may be between

[10] This definition of truth is offered by Beaver (2001: 253) and (van Benthem *et al.* 1997: 594) but differs from Veltman (1996). My preference for this definition is discussed in §4. The observation that classical consequence is a special case of dynamic consequence is my own.

[11] Another Peircian idea. See Wirth (1999) for more on Peirce's abductive theory of interpretation. For a more modern abductive theory of interpretation see Hobbs *et al.* (1993).

that agent's private information and s.[12] This helps highlight that the current picture of indicative conditionals treats them as a species of epistemic modal and regards their function as importantly *social*. They facilitate pooling of information through deference to trusted information sources, without requiring the inefficient and unilateral reproduction of that trusted source's entire information state. It might seem that the upshot is just an obtuse way to communicate $\neg(\phi \wedge \neg\psi)$, but it is not. When embedded under negation, it will not entail $\phi \wedge \neg\psi$, unlike the material conditional. Indeed, that is the principal advantage of a strict analysis over a material one. It is time to examine these logical questions in more detail.

3 The Logic of Indicative Conditionals

Conditional logics themselves are quite complex, and the diversity of approaches to the topic amplify that complexity. All contemporary approaches combine semantic and pragmatic tools to predict the intuitive patterns found in natural language. I will begin by highlighting the patterns I see as central to this debate – focusing on indicative conditionals – and the existing positions in the literature. I will then give an overview of the position I develop in this section, as well as its rationale.

Consider these five entailments of the material conditional ($\supset$):

(3) **False Antecedent (FA)** $\neg\phi \vDash \phi \supset \psi$
Bob didn't dance. So, if Bob danced, he was a turnip.

(4) **True Consequent (TC)** $\psi \vDash \phi \supset \psi$
That coin came up heads. So if it came up tails, it came up heads.

(5) **Material Negation (MN)** $\neg(\phi \supset \psi) \vDash \phi$
It's not true that if God exists, he's a turnip. So, God exists.

(6) **If-And to Or-If (IO)** $(\phi_1 \wedge \phi_2) \supset \psi \vDash (\phi_1 \supset \psi) \vee (\phi_2 \supset \psi)$
If that coin comes up heads and you bet on heads, you will win. So either, (i) if that coin comes up heads, you will win, or (ii) if you bet on heads, you will win.

(7) **Antecedent Persistence (AP)** $\vDash \phi \supset (\psi \supset \phi)$
If that coin came up heads, then that coin came up heads if it came up tails.

Grice (1989 [1967]) and Jackson (1979, 1987) devised means to pragmatically explain away the oddity of FA and TC, thus helping sustain a material conditional analysis. But those techniques are no help with MN, IO and AP, precisely because they are a consequence of how the material conditional behaves when semantically embedded. These problems provide at least some motivation for the classic strict analysis ($\dashv$) of

[12] Willer (2013) develops Beaver's (2001: Ch.8) model of information states to explain how a similar test semantics for *might* could be informative. On that account, an information state S is a set of s's. A may be compatible with some s and thus be compatible with the agent's information. But A that is not to say that it is a live possibility. That requires A to be compatible with every s. The informative effect of Might(A) is to make A a live possibility by eliminating any s's that don't contain a A-world. Can a parallel account of $A \rightarrow B$ work here? The idea would be to eliminate each s containing some $A \wedge \neg B$-worlds. But that will very often eliminate all s's leading to $\varnothing$, thus requiring the kind of pragmatic story I tell here. So while Willer's model nicely captures the informativeness of epistemic possibility claims, it does not help with epistemic necessities. Thus the pragmatic account offered above seems necessary.

Definition 1, since it invalidates FA, TC, MN *and* IO. But this success is little consolation, since AP and variants of FA-MN turn out valid, namely IA and NC:

(7) **Antecedent Preservation (AP)** $\vDash \phi \dashv (\psi \dashv \phi)$
If that coin came up heads, then that coin came up heads if it came up tails.

(8) **Impossible Antecedent (IA)** $\Box\neg\phi \vDash \phi \dashv \psi$
It's impossible that Bob danced. So, if Bob danced, he was a turnip.

(9) **Necessary Consequent (NC)** $\Box\psi \vDash \phi \dashv \psi$
It's impossible that that coin came up heads. So if that coin came up heads, it came up tails.

This failure opens the door for the competing variably-strict account developed by Stalnaker (1968, 1975) and the probabilistic account developed by Adams (1965, 1975). Both accounts invalidate (3)-(9). However, they also *invalidate* many principles which previous logicians had held sacrosanct, namely:[13]

(10) **Import-Export** $\phi_1 \to (\phi_2 \to \psi) \dashv\vDash (\phi_1 \wedge \phi_2) \to \psi$

Antecedent Strengthening (AS) $\phi_1 \to \psi \vDash (\phi_1 \wedge \phi_2) \to \psi$

Simplify Disjunctive Antecedents (SDA) $(\phi_1 \vee \phi_2) \to \psi \vDash (\phi_1 \to \psi) \wedge (\phi_2 \to \psi)$

Transitivity $\phi_1 \to \phi_2, \phi_2 \to \psi \vDash \phi_1 \to \psi$

Contraposition $\phi \to \psi \vDash \neg\psi \to \neg\phi$

Instances of these principles in natural language often sound good. But, with the exception of Import-Export, Adams and Stalnaker offered examples that do not. This is a *prima facie* problem for strict accounts, which typically validate these patterns. Stalnaker (1975) was also able to explain why SDA, Transitivity and Contraposition usually sound good by introducing the notion of a *reasonable inference*; a different, *pragmatic* sense in which these inferences are good. Despite these small surprises, Stalnaker (1968, 1975) and Adams (1965, 1975) validate familiar principles:

(11) **Modus Ponens (MP)** $\phi, \phi \to \psi \vDash \psi$

(12) **Identity** $\vDash \phi \to \phi$

(13) **Modus Tollens (MT)** $\phi \to \psi, \neg\psi \vDash \neg\phi$

Brushing Import-Export quickly under the rug, it looks like Stalnaker (1968, 1975) and Adams (1965, 1975) have won the day, and the real action in conditional logic is deciding between their accounts. The new wave of strict conditional theorists disagree.

Warmbrōd (1983: §5), Veltman (1986, 1985: 186-198) and Gillies (2009: 338, 347) observe a pattern in all the counterexamples to AP, IA, NC, NC, AS, SDA, Transitivity and Contraposition: the conditional conclusion is not felicitous in a context where the premise has been accepted. More specifically, these inferences involve conclusions where an indicative antecedent is not possible with respect to mutual information, a felicity constraint both Stalnaker (1975) and Adams (1975) endorse (examples discussed in §3.1). This observation offers a second life to the strict conditional analysis.

[13] McGee (1989) extends Adams' approach to handle some embeddings and validate import-export. But the resulting account is implausible for other reasons (Edgington 2008: §4.3).

However Warmbrōd (1983: §5), Veltman (1986, 1985: 186-198) and Gillies (2004, 2009) revive the strict analysis in significantly different ways that make different predictions on the relevant details.

- Warmbrōd (1983: §5):
    - *Infelicity*: pragmatic
    - *Semantics*: classic strict conditional (Definition 1)
    - *Logic*: AP, IA, NC, (10), MP, Identity, MT
- Veltman (1986, 1985: 155-197):
    - *Infelicity*: pragmatic
    - *Semantics*: data semantics (close to Definition 3)
    - *Logic*: TC, FA, AP, IA, NC, (10), MP, Identity
- Gillies (2004, 2009):
    - *Infelicity*: maybe pragmatic (2009: 345-6), maybe semantic (2009: 346-8)
    - *Semantics*: either dynamic (Definition 3) or context-shifting (Definition 2)
    - *Logic*: TBD

The important thing to note is that both Warmbrōd (1983) and Veltman (1986, 1985) validate Import-Export and invalidate IO and MN. Those are important achievements. Veltman (1986, 1985) offers compelling and rigorous pragmatic explanations for why FA, TC, IA and NC sound odd. Nonetheless, I will argue against both the Warmbrōd (1983) and Veltman (1986, 1985) analyses. First, I will argue that the infelicity cannot be treated as pragmatic (§3.1), roughly for reasons AP might suggest: sometimes we must consider whether the antecedent of a conditional is felicitous when it is embedded in the consequent of a conditional and so interpreted in a derived context that results from accepting the main antecedent. Second, I show that there are in fact counterexamples to AS, SDA, Transitivity and Contraposition that do not fit the pattern of Stalnaker and Adams' examples, and thus cannot be explained away by pointing to felicity conditions. The first criticism suggests following Gillies (2009: 346-8) and treating the felicity conditions as semantic presuppositions. However, Gillies (2009) left open the question of how an account of logical consequence should count instances with failed presuppositions, and so left open the general logic. It is first observed (§3.2) that a semantic presupposition account together with the standard definition of dynamic consequence (Definition 5), yields literally no logical validities. This motivates a Strawsonian definition of consequence, resulting in the following view:

- *Infelicity*: semantic
- *Semantics*: dynamic, with presupposition (Definition 3)
- *Logic*: TC, FA, AP, IA, NC, Import-Export, MP, Identity
    - Using dynamic, Strawsonian consequence relation

The semantic treatment of felicity conditions is enough to explain away bad instances of AP. Apparent counterexamples to FA, TC, IA and NC can be explained away in basically the way proposed by Veltman (1986, 1985). The fact that the resulting logic does not validate AS, SDA, Transitivity or Contraposition is quite surprising given that it is a strict analysis. But these principles fail in this system in precisely the way you would expect given the counterexamples I raise. The form of those counterexamples provides the key to showing that limited versions of these principles are nonetheless valid, and it is argued that these limited forms exactly capture the intuitively valid instances. Finally, I discuss the striking exclusion of MT. This feature is shared with Veltman (1986, 1985) and counterexamples like his are presented. Like before, it is shown that a restricted form of MT is nevertheless valid and coincides with intuition.

How does the resulting logic compare with Stalnaker (1968, 1975) and Adams (1965, 1975)? Quite favorably, in fact. It improves on them by validating Import-Export, which they cannot even predict to be pragmatically reasonable. It improves on them by explaining why many instances of AP sound good, which they again cannot capture pragmatically. It improves on them in explaining why MT fails in select instances. It might be thought that AS, SDA, Transitivity and Contraposition are a wash, since both accounts can explain why many instances sound good. However, this is not so. I argue in §3.1 that Stalnaker's (1975) pragmatic concept of reasonable inference does not explain generalizations of these patterns that apply within the consequents of conditionals. Further, it incorrectly predicts that my new counterexamples to AS, SDA, Transitivity and Contraposition should be reasonable inferences. I thus conclude that the logic offered here is a significant improvement on those discussed here.

3.1 Pragmatic or Semantic Infelicity, and Why Care?

The relevant felicity constraint on indicative conditionals was illustrated already in (2).

(2) Paula Radcliffe didn't run on Tuesday. #If Paula Radcliffe ran on Tuesday, everyone else lost.

The constraint is simply that the antecedent of an indicative conditional must be possible with respect to what's mutually presupposed in the conversation. The first sentence of (2) makes it mutually presupposed that Paula didn't run, but the indicative that follows requires some worlds where Paula ran. How could this be relevant to the *logic*? As Veltman elegantly puts it:

> Some arguments are logically valid but pragmatically incorrect. Others are pragmatically correct but logically invalid... [U]nfortunately, most of us draw them differently. What one calls a logically valid argument form with a few pragmatically incorrect instances is for another a logically invalid argument form with many pragmatically correct instances. (Veltman 1986: 147)

(2) makes this point clear for FA (and it is easy to see how to extend it to IA). On the basis of examples like (2), many have questioned the validity of FA. But one could just as well treat it as an infelicitous instance of a nonetheless valid pattern. The source of the infelicity can be verified by explicitly adding the felicity condition to the argument. The premise then becomes unacceptable:

(14) For all we know, Paula Radcliffe ran on Tuesday. #Paula Radcliffe didn't run on Tuesday. If Paula Radcliffe ran on Tuesday, everyone else lost.

As Veltman (1986: 165) goes on to point out, the 'counterexamples' to Transitivity offered by authors like Stalnaker (1968, 1975) and Adams (1965, 1975), can also be explained by strict conditional theorists as an instance of a valid pattern which is infelicitous in exactly the same way that (2) is. Consider Adams' (1965) 'counterexample' (15) and the restatement of it containing the conclusion's felicity conditions (16):

(15) If Jones wins the election, Smith will retire to private life. If Smith dies before the election, Jones will win it. So, if Smith dies before the election, he will retire to private life.

(16) For all we know, Smith might die before the election. # If Jones wins the election, Smith will retire to private life. If Smith dies before the election, Jones will win it. So, if Smith dies before the election, he will retire to private life.

The second premise now seems wrong, especially if you are a strict conditional theorist. Smith might die before the election, so Jones could win without Smith subsequently retiring to private life. As Warmbrōd (1983) and Veltman (1986, 1985) detail, exactly the same diagnoses can be given for the alleged counterexamples to TC/NC, AS, SDA and Contraposition. Thus it would seem that there is no advantage, logically, for the Stalnaker (1968, 1975) and Adams (1965, 1975) accounts. Indeed, it might be thought that they are at a disadvantage because they don't explain the many good sounding instances of AS, SDA, Transitivity and Contraposition. However, as the Veltman quote intimates they too can draw on pragmatics. Each of them offer definitions of 'pragmatically reasonable inferences' which predict these patterns to sound good. In doing so, they rely on exactly the same fact: it is pragmatically infelicitous to assert an indicative conditional when its antecedent is ruled out. The debate starts to look like a stand-off, modulo Import-Export. But new insight comes from examining the nature of the infelicity at issue.

Veltman (1986, 1985) was quite clear that the relevant felicity was *pragmatic*. In particular, he takes the felicity condition of indicative conditionals to be a Gricean conversational implicature. Warmbrōd (1983: §) states it as a primitive pragmatic constraint on normal interpretive contexts for conditionals. Either way, the infelicity arises from the assertion of a conditional, and pragmatic rules for interpreting that kind of assertion. This means that the constraint only applies when the conditional is asserted. In this case, the constraint should not apply when the conditional is embedded in the consequent of a conditional. Yet, consider the putative counterexample to a principle of this kind validated by all strict accounts, namely AP:

(17) If that coin came up heads, then that coin came up heads if it came up tails.

What can a strict conditional theorist say about this example? There is no premises, so adding the felicity condition of the antecedent has no impact. Further, the embedded antecedent is not asserted, so talk of its pragmatic felicity is a category mistake. Nonetheless, it is pretty clear what a strict conditional theorist who believes the antecedent shifts the information space would *like* to say. After shifting to the worlds where the coin comes up heads, then the embedded antecedent – *it came up tails* – is no longer possible. So in the context of the consequent, the embedded conditional is not felicitous because its antecedent is not possible in that context. But how do we tell that story without making a category mistake? One can follow Gillies (2009: 346-8) and treat it as a semantic presupposition. As Gillies illustrates this can be done elegantly in the dynamic framework by formulating the constraint as a definedness condition on updates.[14] The modified version of our dynamic strict conditional simply adds to each defined output state that the antecedent is true in some world in s, and that the output is otherwise undefined.[15]

Definition 8 (Presuppositional Dynamic Strict Conditional)

$$s[\phi \to \psi] = \begin{cases} s & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s[\phi] \vDash \psi \\ \varnothing & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s[\phi] \nvDash \psi \\ \text{Undefined} & \text{otherwise} \end{cases}$$

Since only the strict semantics in Definition 8 can explain away putative counterexamples to AP, it seems preferable to the approaches of Veltman (1986, 1985) and Warmbrōd (1983). But that conclusion is premature. No one has shown that this new semantics has the same logic as Veltman (1986, 1985) or Warmbrōd (1983). As a matter of fact, it does not. But before exploring this new logic, I want to draw out another consequence of taking embedded conditionals seriously in logical considerations.

All reasonable logics for conditionals validate the inference from $\phi_1 \to \psi$ and $\phi_2 \to \psi$ to $(\phi_1 \vee \phi_2) \to \psi$. This means that for a theory which validates SDA, the following equivalence holds: $(\phi_1 \vee \phi_2) \to \psi \Dashv \vDash (\phi_1 \to \psi) \wedge (\phi_2 \to \psi)$. Consider then a case where a conditional with a disjunctive antecedent is embedded: $\phi_3 \to ((\phi_1 \vee \phi_2) \to \psi)$. Substitution of equivalents in the consequent allows us one to infer: $\phi_3 \to ((\phi_1 \to \psi) \wedge (\phi_2 \to \psi))$. All reasonable logics also allow weakening of the consequent, yielding $\phi_3 \to (\phi_1 \to \psi)$. Thus, theories which semantically validate SDA can predict that the following inference sounds good:

(18) a. If Gnossos brings tea, then if Heff brings sugar or Peggy brings honey, we'll have a tea party.

b. So if Gnossos brings tea, then if Heff brings sugar, we'll have a tea party.

However, analyses like Stalnaker (1968, 1975) and Adams (1965, 1975) which merely regard SDA as a pragmatically justifiable form of reasoning cannot. Their definitions of reasonable inference only allow one to make the SDA inference when the conditionals in question are themselves asserted. Since similar examples can be constructed for Contraposition, this is a rather significant advantage for accounts like Veltman (1986, 1985) and Warmbrōd (1983) who treat these patterns as logically, not pragmatically, valid. There is a temptation, then, to think that the best logic of indicative conditional will validate SDA, Contraposition and Transitivity. But the situation is more complicated. There are counterexamples to these patterns which have not been observed in the literature.

Consider first AS, which deems the following inference correct:

(19) a. If Gnossos brings tea, Peggy might bring honey.

b. If Gnossos brings tea and Peggy doesn't bring honey, Peggy might bring honey

[14] For more on presupposition in dynamic semantics: Beaver (2001), Heim (1982).

[15] Gillies (2009: 346-8) formulates the constraint differently: $s[\phi] \neq \varnothing$. Only my version captures the fact that an antecedent like $\Diamond A \wedge \neg A$ is infelicitous. Surprisingly, this is essential for validating Identity in full generality (see Fact 14, Appendix A.5.2).

But there is no temptation to embrace (19b) on the basis of (19a). Further, this egregious instance of AS cannot be explained away in the same way as those offered by Stalnaker and Adams. The ability to accept the premise is not at all undermined by accepting that the conclusion's antecedent is possible.

(20) a. Gnossos might bring tea, and Peggy might bring honey. If Gnossos brings tea, Peggy might bring honey.

b. If Gnossos brings tea and Peggy doesn't bring honey, Peggy might bring honey

This is a serious problem for theories like Veltman (1986, 1985) and Warmbrōd (1983). But it is also a problem for Stalnaker (1975), since it is predicted to be a reasonable inference.[16] A similar counterexample can be constructed to SDA. We need some money, and we bet heads on a coin flip. But we don't know whether the coin flip happened. Our coin flipper is a bit flaky. I can optimistically assert (21a), but there is no temptation to infer (21b).

(21) a. If the coin came up heads or tails, maybe it came up heads.

b. If the coin came up tails, maybe it came up heads.

Furthermore, our commitment to the premise is unfazed by adding that the conclusion's antecedent is possible.

(22) a. Maybe the coin came up tails. But, if the coin came up heads or tails, maybe it came up heads.

b. If the coin came up tails, maybe it came up heads.

Again, this is a serious problem for strict and variably-strict theorist alike. Strict theorists predict this instance to be valid and pragmatically felicitous and variably-strict theorists predict it to be pragmatically reasonable. Transitivity falls prey to the same sort of example.There is no temptation to infer (23b) from (23a).

(23) a. If Paula didn't run, she is sick. But if she's sick, maybe she (still) ran.

b. If Paula didn't run, maybe she (still) ran.

As in the other cases, adding the presupposed material to the premises does not impact their acceptability.

(24) a. Maybe Paula didn't run. Maybe she's sick. If Paula didn't run, she is sick. But if she's sick, maybe she (still) ran.

b. If Paula didn't run, maybe she (still) ran.

[16] Perhaps Stalnaker would treat both (19a) and (19b) on the model of his analysis of *might* counterfactuals, where an epistemic modal is taken to scope over the whole conditional (Stalnaker 1984: 143). Then the conditionals are embedded under *might*, which would block the pragmatic reasoning of reasonable inference. But this strategy is unpromising. First, that treatment of *might* conditionals cannot capture mixed consequents as in *If a is red, then a might be a square and must be large.* This is clearly not equivalent to *It might be that, if a is red, then it is a square and it is large.* Second, one can construct good instances of AS containing a *might*-consequent that one would like to predict: *if a might be red, then a might be red; so if a might be red and a might be blue, then a might be red.*

Parallel examples for AP, TC and NC exist, but I trust the reader has grasped the basic pattern here. All of these examples involve epistemic *might* or *maybe* in the consequents. Veltman (1986, 1985) and Warmbrōd (1983) explicitly aimed to include operators of these kinds in their logical systems, so it is not shifting the domain of inquiry. While there is some question of how exactly Stalnaker (1968, 1975) or Adams (1965, 1975) would even approach these examples, it is clear that there is no good reason to simply exclude them from our logical inquiry. And, there is good reason to think they would need to predict them as reasonable inferences. So it would seem everyone has some work to do.[17] Considering conditionals with modals in their consequents also furnishes counterexamples to Contraposition and Modus Tollens.

Take Contraposition. Suppose we're in a windowless room and have no information about whether it's raining. We are still in a position to know (25a). Yet, I am reluctant to infer (25b).

(25) a. If it's pouring, it must be raining.
b. If it might not be raining, it's not pouring.

But assuming that *not must* is equivalent to *might not*, this is an instance of contraposition. And, as before, adding the presupposition of the conclusion to the premise doesn't make the premise any less acceptable.

(26) a. For all we know, it might not be raining. But, if it's pouring, it must be raining.
b. If it might not be raining, it's not pouring.

So again, there is a problem for all of our candidate analyses. Of course, Contraposition is closely related to the principle of Modus Tollens. So it is not surprising that we have a failure of MT in exactly the same kind of scenario.[18]

(27) a. It might not be raining. But, if it's pouring, it must be raining.
b. So, it's not pouring.

In fact, Veltman (1986, 1985: 158) already observed examples of these kinds in defending the fact that his logic invalidates MT.[19] But it is a deeply worrying kind of example for Warmbrōd (1983), Stalnaker (1975) and Adams (1975) who all validate MT. Indeed, Adams (1988) worried about some putative counterexamples to MT:

(28) a. If it rained, it didn't rain hard. It *did* rain hard.
b. So, it didn't rain

Unlike (30), the second premise cannot be placed first:

(29) a. # It rained hard. If it rained, it didn't rain hard.

[17] It is an interesting question how a Kratzerian (1986; 1991; 2012) analysis of modals and conditionals would treat these examples. However, there is no existing logical study of such analyses, and the basic theory can be used to formulate strict and variably-strict accounts alike by selecting the right constraints on the modal base and ordering source. As far as I can tell, the Kratzerian articulations of the strict and variably-strict views would face the same dilemma, but much more investigation here is needed.

[18] I again assume *might* and *must* are duals, but you could read the first premise as *It's not true that it must be raining.*

[19] Yalcin (2012) defends probabilistic versions of these counterexamples.

b. So, it didn't rain

On reflection, it seems clear then that Adams' (1988) premises are either contradictory or lead to a pragmatically defective context. No such diagnosis works for example (30).

I hope the dramatic tension is at its peak. Many instances of AS, SDA, Transitivity, Contraposition and Modus Tollens sound impeccable. Strict theories that validate these principles cannot pragmatically explain away these new counterexamples. But variably-strict and probabilistic theories which invalidate them have to pragmatically explain why these principles often sound so good, and those explanations appear to predict that these new counterexamples should be among those good pragmatic inferences. But recall the earlier progress made in this section. It was seen that the felicity conditions of indicative conditionals should be treated as semantic presuppositions, and amended the dynamic strict analysis from §2 to suit. In the next section, I will show something rather surprising: when equipped with the right notion of consequence, the dynamic strict analysis can resolve the tension. It validates a principled restriction of AS, SDA, Transitivity, Contraposition and MT, as well as delivering on the other criteria discussed at the outset of §3.

3.2 Dynamic Strict Conditional Logic

This section will explore the logic of the following semantics.

Definition 8 (Presuppositional Dynamic Strict Conditional)

$$s[\phi \to \psi] = \begin{cases} s & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s[\phi] \vDash \psi \\ \varnothing & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s[\phi] \nvDash \psi \\ \text{Undefined} & \text{otherwise} \end{cases}$$

Of course a semantics must be paired with a definition of consequence to generate a logic. The dynamic definition of consequence from §2 was this.

Definition 5 (Dynamic Consequence)

$$\phi_1, \ldots, \phi_n \vDash \psi \iff \forall s\colon s[\phi_1]\cdots[\phi_n] \vDash \psi$$

The basic idea was that updating any state with the premises will lead to a state which supports the conclusion. However, Definition 8 has introduced a new feature to attend to, namely the fact that updates can be undefined. Indeed, briefly reflecting on this feature illustrates the inadequacy of Definition 5. Consider any logical principle, for example Identity. There will be an instance of it with a contradictory antecedent, e.g. $(\mathsf{A} \wedge \neg\mathsf{A}) \to (\mathsf{A} \wedge \neg\mathsf{A})$. But then $s[(\mathsf{A} \wedge \neg\mathsf{A}) \to (\mathsf{A} \wedge \neg\mathsf{A})]$ will be undefined for any s. That means it will not be the case that $s \vDash (\mathsf{A} \wedge \neg\mathsf{A}) \to (\mathsf{A} \wedge \neg\mathsf{A})$ since that requires $s[(\mathsf{A} \wedge \neg\mathsf{A}) \to (\mathsf{A} \wedge \neg\mathsf{A})] = s$, and $s[(\mathsf{A} \wedge \neg\mathsf{A}) \to (\mathsf{A} \wedge \neg\mathsf{A})]$ is undefined, so it can't be identical to s. Modus Ponens will also fail because of an instance with premises $\mathsf{A} \wedge \neg\mathsf{A}, (\mathsf{A} \wedge \neg\mathsf{A}) \to \mathsf{B}$ will lead to undefinedness. However, it's pretty clear what needs to be done. The definition should be modified so as to only count instances where presuppositions are met. Indeed, Strawson's (1952: 173-9) work on universal quantification advocated for a consequence definition of this kind, and others have utilized it as well (von Fintel 1999; Beaver 2001). The needed modification comes to this:

Definition 9 (Strawsonian Dynamic Consequence)

$$\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall s\colon \text{if } s[\phi_1]\cdots[\phi_n][\psi] \text{ is defined, } s[\phi_1]\cdots[\phi_n] \vDash \psi$$

This definition requires the exact same relation to hold, it just adds the constraint that sequentially updating with the premises and conclusion leads to a defined state. Many of the interesting inference patterns involve other connectives, so it will be necessary to have a semantics for them. I will adopt fairly standard definitions for them in update semantics (Groenendijk *et al.* 1996; Veltman 1996), namely those given in Definition 10. For convenience, worlds are treated as functions from atomics to truth-values. Atomic sentences update s by eliminating worlds where they are false, i.e. by keeping only those where $w(A) = 1$. Negation works by eliminating the worlds that would satisfy an update with the negated formula. Conjunction is sequential update with each conjunct. Disjunction forms the union of the separate updates of s with each disjunct.

Definition 10 (Update Semantics)

(1)	$s[A]$	$=$	$\{w \in s \mid w(A) = 1\}$
(2)	$s[\neg\phi]$	$=$	$s - s[\phi]$
(3)	$s[\phi \wedge \psi]$	$=$	$(s[\phi])[\psi]$
(4)	$s[\phi \vee \psi]$	$=$	$s[\phi] \cup s[\psi]$
(5)	$s[\Diamond\phi]$	$=$	$\{w \in s \mid s[\phi] \neq \varnothing\}$
(6)	$s[\Box\phi]$	$=$	$\{w \in s \mid s[\phi] = s\}$

Just like conditionals, epistemic possibility and necessity are tests. $\Diamond\phi$, which approximates *Might* ϕ, tests that there is at least one ϕ-world in s. By contrast, $\Box\phi$, which approximates *Must* ϕ, tests that *all* the worlds in s are ϕ-worlds.[20] I am now in a position to describe the logic that results from these definitions.

Because the definition of consequence discounts cases of failed presupposition, you might think it would straightforwardly validate standard strict-conditional principles like AS, SDA, Transitivity and Contraposition. After all, that would be the prediction of Warmbrōd and Veltman's theories if modified in this way. However, these principles fail in exactly the way you would expect them to given the counterexamples discussed at the end of §3.1: AS, SDA and Transitivity fail when the conclusion has a consequent like $\Diamond\psi$; Contraposition and MT fail when the premise has a consequent like $\Box\psi$. Why is this? These operators have unique features behind this behavior. $\Diamond\phi$ is **not** *persistent*: a state s can support $\Diamond\phi$ while a more informed version of that state s', i.e. $s' \subset s$, does not support $\Diamond\phi$.

Definition 11 (Persistence) ϕ is *persistent* iff $s' \vDash \phi$ if $s \vDash \phi$ and $s' \subseteq s$. (Support for ϕ persists after more information comes in.)

Just consider a simple example where $s = \{w_0, w_1\}$, where $w_0(\mathsf{A}) = 0$ and $w_1(\mathsf{A}) = 1$.[21] s will support $\Diamond\mathsf{A}$, but the more informed substate $s' = \{w_0\}$ will not. Intuitively, this just captures the fact that you can think it might be raining, gain more information and no longer think that it might be raining. $\Box\phi$ and $\phi \to \psi$, on the other hand, are **not** *miserly*: a state s can come to support $\Box\phi$ after gaining more information, even though s does not initially support $\Box\phi$.[22]

Definition 12 (Miserly) ϕ is *miserly* iff $s' \nvDash \phi$ if $s \nvDash \phi$ and $s' \subseteq s$.
(s continues to withhold support of ϕ even after s is enriched with more information.)

[20] This semantics of epistemic modals originates in Veltman (1996). See also Van der Does *et al.* (1997) and Willer (2013) for discussion and extensions.
[21] This terminology comes from Groenendijk *et al.* (1996: §4.2.2). Veltman (1985, 1986) calls it T-stability.
[22] Veltman (1985, 1986) calls it F-stability.

The fact that $\Box\phi$ is not miserly tracks the fact that ϕ may not be necessitated by your initial information, but later be necessitated once you get more information, i.e. rule out more worlds. The same is true of $\phi \to \psi$ which essentially requires that all the ϕ-worlds are ψ-worlds. That may fail in a space of worlds, but hold in a subset of it. Isolating these semantic features allows one to prove limited versions of the principles discussed above.

Fact 1 If the main consequent is persistent, AS, SDA, Transitivity, AP, TC and NC are valid. (See Appendix A.5.2 for proof.)

Fact 2 If the main consequent is miserly, Contraposition and MT are valid. (See Appendix A.5.2 for proof.)

Just to see the intuitive idea at work here, consider the formalized version of the counterexample to AS:

(19) a. If Gnossos brings tea, Peggy might bring honey.
$\mathsf{T} \to \Diamond\mathsf{H}$

b. If Gnossos brings tea and Peggy doesn't bring honey, Peggy might bring honey. $(\mathsf{T} \wedge \neg\mathsf{H}) \to \Diamond\mathsf{H}$

The reason the conclusion doesn't follow is because its antecedent shifts to the $\mathsf{T} \wedge \neg\mathsf{H}$-worlds and then tests that $\Diamond\mathsf{H}$, which will obviously fail. But the first premise guarantees that if you shift just to the T-worlds that same test will pass. So the key feature here really is the non-persistence of $\Diamond\phi$: that's what allows the first premise to be supported without the second one being supported. A careful examination of the other examples relevant to persistence reveal exactly the same situation behind the scenes. A quick look at the MT example is also instructive.

(30) a. It might not be raining. But, if it's pouring, it must be raining. $\Diamond\neg\mathsf{R}, \mathsf{P} \to \Box\mathsf{R}$

b. So, it's not pouring. $\neg\mathsf{P}$

The conditional premise is supported because when you zoom in on the P-worlds, all of them are R-worlds. But when $\Diamond\neg\mathsf{R}$ is supported that means that, zoomed out, there are some $\neg\mathsf{R}$-worlds. This in no way implies that, zoomed out, there are no $\neg\mathsf{P}$-worlds. Here, it is the fact that $\Box\mathsf{R}$ is not miserly that makes the example possible. Zoomed out, $\Box\mathsf{R}$ is not supported. But zoomed in to the P-worlds, it is.

It will have been noted that limited versions of AP, TC and NC are validated. Intuitive counterexamples with $\Diamond$ in the consequent exist for these patterns counterexamples which cannot be dispelled by the usual attention to the antecedent's presupposition (see Remarks 10 and 11 in Appendix A.5.2). However, all of the traditional counterexamples to these patterns can be dispelled in this way. This was already discussed for AP in §3.1. For discussion of TC/NC, see Veltman (1986: §4.2).

As for unrestricted logical principles, the following summarizes the highlights.

Fact 3 Modus Ponens, Identity, Import-Export and the Deduction Equivalence ($\phi \vDash \psi \iff \vDash \phi \to \psi$) hold in full generality. (See Appendix A.5.2 for proof.)

The fact that Import-Export holds is pretty easy to see given that conjunction is treated as sequential update. $s[(\phi_1 \wedge \phi_2) \to \psi] = s$ just in case $s[\phi_1][\phi_2] \vDash \psi$. But $s[\phi_1 \to (\phi_2 \to \psi)] = s$ just in case $s[\phi_1] \vDash \phi_2 \to \psi$, which is just to say $s[\phi_1][\phi_2] \vDash \psi$.

Like other strict accounts, the dynamic one invalidates problematic patterns that are impossible to pragmatically explain away on a material conditional analysis. MN was a case in point. MN would require $s[\neg(\mathsf{A} \to \mathsf{B})] = s[\neg(\mathsf{A} \to \mathsf{B})][\mathsf{A}]$. Intuitively, some $\mathsf{A} \wedge \neg\mathsf{B}$-world is enough for $\mathsf{A} \to \mathsf{B}$ to fail, but having one of those around certainly doesn't require that all the worlds in s are A-worlds. The semantics predicts this exactly. Suppose w_1 is such a $\mathsf{A} \wedge \neg\mathsf{B}$-world, and w_0 is a $\neg\mathsf{A}$-world. Letting $s = \{w_0, w_1\}$, the semantics of negation tells us that $s[\neg(\mathsf{A} \to \mathsf{B})] = s - s[\mathsf{A} \to \mathsf{B}]$. $s[\mathsf{A} \to \mathsf{B}]$ tests that $\{w_0, w_1\}[\mathsf{A}] \vDash \mathsf{B}$. But $\{w_0, w_1\}[\mathsf{A}] = \{w_1\}$ and $\{w_1\} \nvDash \mathsf{B}$. So $s[\mathsf{A} \to \mathsf{B}] = \varnothing$, meaning $s[\neg(\mathsf{A} \to \mathsf{B})] = s - \varnothing = s$. So MN's requirement comes to $s = s[\mathsf{A}]$. But this isn't true since s contains the $\neg\mathsf{A}$-world w_0. Similarly IO – $(\phi_1 \wedge \phi_2) \supset \psi \vDash (\phi_1 \supset \psi) \vee (\phi_2 \supset \psi)$ – is not valid. The premise requires all the $\phi_1 \wedge \phi_2$-worlds to be ψ worlds. But that certainly doesn't require either that all the ϕ_1-worlds be ψ-worlds, or that all the ϕ_2-worlds be ψ-worlds. However, the semantics does predict that FA/IA is valid. But careful examination of this prediction assuages any worries.

FA – $\neg\phi \vDash \phi \to \psi$ – is a case where the presuppositions are *never* met, and it therefore vacuously satisfies Definition 9. But even if one grants the technical validity of FA, the semantics above does not predict than any instance of FA should sound acceptable in any context to competent speakers. After all, acceptability requires presuppositions to be met and FA cannot meet them in any context. Accordingly, one can grant vacuous validities to simplify the logic without predicting that they govern the intuitions of any speaker in any context. (IA is just a special instance of FA, so the same reasoning applies there.)

This section has articulated a new form and rationale for a dynamic strict conditional semantics. I have argued that when properly developed, this kind of analysis can better capture the logic of natural language indicative conditionals. As it has now become clear, this semantics is primarily concerned with which formulas are supported by some given information, and which inferences preserve the flow of information from premises to conclusion. Veltman (1985, 1986) and Gillies (2009) gloss the idea of some information state s supporting ϕ as ϕ being true according to s. But many philosophers might regard this way of thinking about truth-conditions as insufficiently objective. Criticisms along these lines are voiced against Veltman (1985, 1986) by Adams (1986). In the next section, I elaborate on what the definition of truth offered in §2 predicts about the truth-conditions of indicative conditionals. It will become clear there that a familiar correspondence conception of truth is possible, and has some useful applications in the semantics of conditionals.

4 The Truth Conditions of Indicative Conditionals

Here, I wish to explain how the semantics above endows indicative conditionals with partial truth-conditions, why that is an interesting view, and how it improves on previous attempts to develop that view. The partial truth-conditions are a direct consequence of the semantics in Definition 9 and the general definition of truth.

Definition 6 (Truth) $w \vDash \phi \iff \{w\}[\phi] = \{w\}$

Fact 4 (Truth-Conditions for Presupp. Dynamic Strict Conditionals)

1. $\phi \to \psi$ is true in w, if both ϕ and ψ are true in w.

2. $\phi \to \psi$ is false in w, if ϕ is true in w and ψ is false in w

3. Otherwise, $\phi \rightarrow \psi$'s truth-value is undefined in w

The important part is that when ϕ is false in w, $\{w\}[\phi] = \varnothing$ in which case $\nexists w' \in \{w\}: w' \in \{w\}[\phi]$. But that is just to say that the presupposition of the conditional is not met. Thus, an update with the conditional will be undefined in $\{w\}$. In that case we can neither say that $w \vDash \phi \rightarrow \psi$ nor $w \vDash \phi \rightarrow \psi$.

I've just offered truth-conditions for indicative conditionals, but why bother? One reason is simply to satisfy a philosophical impulse. It might be that the conversational dynamics and logic of indicative conditionals goes by support rather than truth. But one might still want to say what they represent the world as being like. The notion of truth encoded in Definition 6 serves this role admirably. As d'Alembert (1995: 29 [1751]) wrote: "The universe... would only be one fact and one great truth for whoever knew how to embrace it from a single point of view." According to the analysis of indicative conditionals offered above, their use in our everyday speech and reasoning rely essentially on a perspective of uncertainty. This is *why* their truth-conditions do not figure prominently the in best explanation of those activities: truth-conditions are only visible from an omniscient perspective. Some may object that this results in an error theory of ordinary uses of *true* and *false*. Ordinary speakers do routinely employ those terms in situations where the theory above says support is in fact operable. However, it is an unstated assumption that these ordinary uses need to be analyzed as being identical to the theoretical concepts of truth and falsity developed by semanticists and philosophers. Furthermore, trying to systematize those ordinary uses using only the theoretical conception of truth is quite tricky. Consider Stalnaker's truth-conditional analysis. There, the truth of an indicative conditional is relativized to a world w and the context set c. Suppose it's taken for granted in the conversation that all the red marbles in the bag are large, and one of us asserts: *if x is a red marble, x is large.* Now, qua ordinary English speaker, I have a question for you: is what was asserted true in a world w' that's incompatible with what we're assuming because in w' many of the red marbles are tiny? The ordinary speakers I've posed with this question typically shrug and try to change the subject. The more persistent ones inevitably note that if you are being really strict, what was said was false, but, on the other hand what was said was perfectly legitimate given what was being assumed. Familiar truth-conditional approaches do not predict this case to be in any way puzzling: the semantics determines a set of worlds, given c and w' either is or isn't in that set – whether or not $w' \in c$ is completely irrelevant. By contrast, the approach above suggests you have offered the ordinary speaker two informational perspectives, c and w' and asked them something which is ambiguous between whether the conditional is supported by c or supported by, i.e. true in, w'. So the theory offered above does not seem to be revisionary at all. Indeed, it seems to be better positioned to capture those cases where the theoretical concepts of truth and falsity preferred by semanticists diverge from our intuitive ones.

While support is definitely the core semantic concept used above, I am not claiming that truth-conditions are to be banned from the tavern. After all, truth-conditions are recoverable, on demand, from the semantics offered above. I now wish to focus on this feature and show how the truth-conditions predicted in Fact 4 can be exploited to solve a puzzle about the interaction of conditionals and adverbs. The essence of these puzzles was first introduced by Lewis (1975), but their extension to modal adverbs was championed by Kratzer (1986). It is well known that there is no single connective which can be used to translate these three sentences containing nominal quantifiers:

(i) *Some As are Bs*, (ii) *All As are Bs* and (iii)*Most As are Bs.* Conjunction is called for in (i), a conditional in (ii) and, well, it's not clear in (iii). This isn't so bad, since there's no real linguistic reason to force the same connective into all of these sentences. However, with modal and temporal adverbs you get these same forms of quantification, explicit conditionals, yet meanings for the existential and proportional quantifiers that resist representation with conditionals. Consider first the modalized conditionals in (31), in the context of trying to guess the colors and sizes of some marbles in a bag.

(31) a. If x is red, x might be large.

b. It might be that x is red and large.

These guesses seem to come to the same thing. To predict this, it would seem that conditionals have to mean *and* in these contexts. Turning to *must*, all of these guesses seem to come to the same thing:

(32) a. It must be that if x is red, x is large.

b. If x is red, x must be large.

c. If x is red, x is large

While the material implication analysis does nicely for (32a), it is quite clearly wrong for (32b). It's unclear how any traditional connective could predict that *must* can just be left out. Further, neither conjunction nor material implication are remotely plausible for (33).

(33) a. Probably, if x is red, x is large

b. If x is red, x is probably large

First, (33a) and (33b) seem equivalent. Second, neither conjunction nor material implication would do well here. A low probability or the falsity of *x is red* does not suffice to make either sentence true. Further, it might appear that any semantics which renders (33a) and (33b) equivalent falls afoul of triviality results which show that the probability of a conditional proposition cannot equal the probability of the consequent given the antecedent (Lewis 1976, 1986; Hájek 1989; Hájek & Hall 1994).

As Gillies (2010) shows, a dynamic strict semantics can explain the equivalences in (31) and (32). As shown in Appendix A.5.2:[23]

Fact 5 (Might) $\phi \to \Diamond\psi \dashv\vDash \Diamond(\phi \wedge \psi)$

Fact 6 (Must) $\Box\phi \to \psi \dashv\vDash \phi \to \psi \dashv\vDash \phi \to \Box\psi$

[23]You might expect $\Diamond(\phi \to \psi)$ to also be equivalent. After all, it seems like another way of saying (31a) is *It might be the case that if x is red, x is large.* However, $\Diamond(\phi \to \psi)$ is supported just in case $\phi \to \psi$ is supported. This is something of a problem for this analysis. I suggest that the wide-scope *might* means *It might be true that.* We could introduce a truth-predicate in our object language to represent this sentence as $\Diamond\mathsf{T}(\phi)$ with the following semantics: $s[\mathsf{T}(\phi)] = \{w \in s \mid w \vDash \phi\}$. Since the conditional is true just in case $\phi \wedge \psi$ is true, this will render $\Diamond\mathsf{T}(\phi \to \psi)$ equivalent to $\Diamond(\phi \wedge \psi)$ and $\phi \to \Diamond\psi$. However, this does not work well for $\Box\mathsf{T}(\phi \to \psi)$. But it could be made to work by modifying $\Box\phi$ to test that $s'[\phi] = s'$ where s' is the set of worlds in s for which ϕ's truth is defined. This would end up making all the wide-scope modals quite like my analysis of *probably* in Definition 14 that is designed to capture wide-scoping. Perhaps then one could go for something closer to Definition 13.2 and hold that all wide-scope modals contain T. The result would be a little less tidy, but perhaps more general.

However, Gillies (2010) stops short of offering a semantics for *probably* that predicts the pattern in (33) or even assigns it an intuitively plausible meaning. As it turns out, this is not a trivial task, and the partial truth-conditions highlighted above become quite useful here. This is no surprise, since Lewis (1975: n14) himself noted that trivalent truth-conditions would provide a way out of his puzzle, and it has been known since de Finetti (1936: 35) that trivalent propositions are a natural fit with conditional probabilities. What is surprising is that these trivalent truth-conditions come for free with the right strict conditional story. This is also nice since the woes of traditional trivalent accounts are a sad tale indeed (§4.2).

4.1 'Probably', Probably

Suppose we wanted to add a modal operator to our language to model (33). A natural way to do so is developed by Yalcin (2012: 1020). Instead of thinking of formulas as just drawing on and updating a space of worlds s, think of them as also drawing on a probability function Pr.[24] Thus states are now a pair $\langle s, Pr\rangle$, written s_{Pr}, where it is assumed $Pr(s) = 1$.[25] To ensure that $Pr(s') = 1$ when s is updated to s', Yalcin conditionalizes Pr on each update. This is illustrated in the modified semantics for atomic sentences in Definition 13.1, where the original probability function is conditionalized so that every proposition x is conditionalized on the new information, namely the A-worlds from s: $\{w \in s \mid w(A) = 1\}$.

Definition 13 (Atomics, Probably and Strict Conditional v1)

1. $$s_{Pr}[A] = \langle \{w \in s \mid w(A) = 1\}, Pr(x \mid \{w \in s \mid w(A) = 1\})\rangle$$

2. $$s_{Pr}[\triangle\phi] = \begin{cases} s_{Pr} & \text{if } Pr(s_{Pr}[\phi]) > 0.5 \\ \varnothing_{Pr} & \text{otherwise} \end{cases}$$

3. $$s_{Pr}[\phi \to \psi] = \begin{cases} s_{Pr} & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s_{Pr}[\phi] \vDash \psi \\ \varnothing & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s_{Pr}[\phi] \nvDash \psi \\ \text{Undefined} & \text{otherwise} \end{cases}$$

The semantics for $\triangle\phi$ construes *probably* as testing that the ϕ-worlds in s have probability greater than 0.5. The crucial feature to note is that conditionals test their consequent in a state that has been updated with the antecedent. Thus, they will test it against a probability function that has been conditionalized on the antecedent. This semantics captures nicely the meaning of $\phi \to \triangle\psi$: within the ϕ-worlds, ψ is probable. This is the kind of conditional that is of interest to Yalcin (2012), but here we must ask whether it delivers a plausible semantics for $\triangle(\phi \to \psi)$. This formula is supported when $Pr(s_{Pr}[\phi \to \psi]) > 0.5$. But $s_{Pr}[\phi \to \psi]$ is either s_{Pr}, in which case it has probability 1, or it is $\varnothing_{Pr}$ in which case it has probability 0. Thus, this semantics predicts that $\triangle(\phi \to \psi)$ is supported exactly when $\phi \to \psi$ is. This makes $\phi \to \triangle\psi$ and $\triangle(\phi \to \psi)$ nonequivalent and gives the latter incorrect support conditions. It seems clear that $\triangle(\phi \to \psi)$ is weaker than $\phi \to \psi$. Suppose there are eight red marbles in the bag and two blue ones. I'd happily assert (34a), but not (34b).

[24] $Pr : \mathcal{A} \mapsto [0,1]$, where $\mathcal{A}$ is a Boolean Algebra on $\wp(W)$.

[25] As well as $Pr(A \cup B) = Pr(A) + Pr(B)$, where $A \cap B = \varnothing$.

(34) a. Probably, if you pick a marble from the bag, it will be red.
b. If you pick a marble from the bag, it will be red.

Now consider a semantics which might seem to be equivalent. Instead of assessing the probability of s updated with ϕ, assess the probability of the worlds in s where ϕ is true, given that it has a truth-value. It comes to the same thing for non-conditional ϕ. Since they always have a truth-value, this conditionalization will idle.

Definition 14 (Probably v2)

$$s_{Pr}[\triangle\phi] = \begin{cases} s_{Pr} & \text{if } Pr(\{w \in s\colon w \vDash \phi\} \mid \{w \in s\colon w \vDash \phi \text{ or } w \nvDash \phi\}) > 0.5 \\ \varnothing_{Pr} & \text{otherwise} \end{cases}$$

This semantics predicts the same behavior for $\phi \to \triangle\psi$, but gives different and more plausible results for $\triangle(\phi \to \psi)$. Instead of assessing s or $\varnothing$ for their probability, it will assess the probability that ϕ is true given that it has a truth-value. This is no surprise given that the definition above parallels the definition of conditional probability:

$$Pr(B \mid A) := \frac{Pr(B \cap A)}{Pr(A)}$$

For a trivalent conditional, the probability of it having a truth-value is the probability of its antecedent, and the probability of it being true is the probability of the antecedent and consequent being true. It is no surprise then that the resulting semantics is able to make the probability of a conditional go along with the probability of the consequent given the antecedent. While I will not endeavor to prove this equivalence here, I will illustrate it with a simple example. Consider a state s_{Pr}, where the worlds it contains and Pr are as given in Table 1. A has a relatively low credence as does $\mathsf{A} \wedge \mathsf{B}$. But, when that low credence in A is split across $\mathsf{A} \wedge \mathsf{B}$ and $\mathsf{A} \wedge \neg\mathsf{B}$ most of it goes to the former. Demonstrating that $\mathsf{A} \to \triangle\mathsf{B}$ and $\triangle(\mathsf{A} \to \mathsf{B})$ come to the same in this setting also helps illustrate how they both differ from $\triangle(\mathsf{A} \wedge \mathsf{B})$. Consider first $\mathsf{A} \to \triangle\mathsf{B}$ in s_{Pr}. Intuitively,

s	A	B
w_0	1	1
w_1	1	0
w_2	0	1
w_3	0	0

		Pr
$s \cap [\![\mathsf{A}]\!]$	$\{w_0, w_1\}$	0.3
$s \cap [\![\neg\mathsf{A}]\!]$	$\{w_2, w_3\}$	0.7
$s \cap [\![\mathsf{A} \wedge \mathsf{B}]\!]$	$\{w_0\}$	0.2
$s \cap [\![\mathsf{A} \wedge \neg\mathsf{B}]\!]$	$\{w_1\}$	0.1

Table 1: Details of the Example

this seems like it should be supported. Among the A-worlds, B is indeed probable. The semantics requires testing whether $s_{Pr}[\mathsf{A}] \vDash \triangle\mathsf{B}$. This involves first conditionalizing on the A-worlds: $\{w_0, w_1\}$. Call this probability function Pr_A. We now check whether, in the shifted state, the test imposed by $\triangle\mathsf{B}$ is passed. That test comes to (35).

(35) $$Pr_A(\{w_0\} \mid \{w_0, w_1\}) > 0.5$$

$$\frac{Pr_A(\{w_0\} \cap \{w_0, w_1\})}{Pr_A(\{w_0, w_1\})} > 0.5 \qquad \text{Def. of } Pr(B \mid A)$$

(36) $$\frac{Pr_A(\{w_0\})}{Pr_A(\{w_0, w_1\})} > 0.5 \qquad \text{Def. of } \cap$$

To determine this, we need to find out what $Pr_A(\{w_0\})$ and $Pr_A(\{w_0, w_1\})$ are.

$$(37) \qquad Pr_A(\{w_0\}) = Pr(\{w_0\} \mid s \cap [\![\mathsf{A}]\!]) \qquad \text{Def. of } Pr_A$$

$$(38) \qquad = Pr(\{w_0\} \mid \{w_0, w_1\})$$

$$= \frac{Pr(\{w_0\} \cap \{w_0, w_1\})}{Pr(\{w_0, w_1\})}$$

$$(39) \qquad = \frac{Pr(\{w_0\})}{Pr(\{w_0, w_1\})} \qquad = \frac{2}{3}$$

It should be pretty clear from this that $Pr_A(\{w_0, w_1\}) = Pr(\{w_0, w_1\} \mid \{w_0, w_1\})$, which is of course 1. Thus (36) amounts to dividing $\frac{2}{3}$ by 1, and asking whether the result is greater than 0.5, as it clearly is.

Evaluating $\triangle(\mathsf{A} \to \mathsf{B})$ in s_{Pr}, requires testing whether the probability of the conditional being true, given that it has a truth-value, is greater than 0.5. The worlds where the conditional $\mathsf{A} \to \mathsf{B}$ is true are $\{w_0\}$, and those where it has a truth-value are $\{w_0, w_1\}$. So the test imposed by $\triangle(\mathsf{A} \to \mathsf{B})$ comes to this:

$$(40) \qquad Pr(\{w_0\} \mid \{w_0, w_1\}) > 0.5$$

As was shown in (38)-(39) above, the left-hand side comes to $\frac{2}{3}$, so this test is passed. Note that this would not be the case on the original semantics in Definition 13.2. The test imposed by $\mathsf{A} \to \mathsf{B}$ in s_{Pr} would not be passed because of w_1, returning $\varnothing$. The probability of $\varnothing$ is decidedly less than 0.5.

This discussion of *probably* serves to illustrate that the partial truth-conditions assigned by the presuppositional dynamic strict analysis are useful in compositional semantics. The particulars of the analysis above are original. But the observation that partial truth-conditions provide a helpful tool for analyzing the interaction of quantifiers, adverbs and conditionals is an old one. It goes back to at least Lewis (1975: n14). Indeed, as Milne (1997) documents, the idea of using trivalent propositions to serve as the objects of conditional probability goes back to de Finetti (1936: 35). It would be wrong to suggest that all of the puzzles surrounding quantifiers, adverbs/modals and conditionals are obviously solved by the account above.[26] But Huitink (2008: Ch.5) has recently advanced a view of this kind, and there is room for optimism. However, the major stumbling block for this kind of view has been in generating a plausible logic of indicative conditionals. In the next section, I summarize these difficulties, the improvements Huitink (2008: Ch.5) has made, and the reason why those improvements are not enough. I conclude that the analysis presented in this paper provides the best method of using partial truth-conditions to analyze the interaction of quantifiers, conditionals and adverbs.

4.2 The Woes of Traditional Trivalence

Many have advocated for the view that indicative conditionals have the partial truth-conditions described earlier. Those who wish to simply give a probabilistic semantics

[26] For some of the puzzles regarding nominal quantifiers see Higginbotham (1986, 2003); von Fintel & Iatridou (2002). For a charge that Gillies' (2010) analysis cannot handle proportional temporal quantifiers see Khoo (2011). I believe the analysis above, equipped with dynamic generalized quantifiers (van den Berg 1996), could solve these puzzles. (Note that the dynamic theory has no need for explicit universal quantification in conditionals or Closed modal spaces, the two features central to Khoo's criticism.) But that story must be saved for another occasion.

are typically inclined to this view (Adams 1965, 1975; McGee 1989; Bennett 2003; Edgington 2008), though it is something of an open question how such a view would follow from their probabilistic semantics. But many have taken the idea farther, and tried to develop this idea as a three-valued logic (Jeffrey 1963; Belnap 1973; Manor 1974; McDermott 1996; Milne 1997). This is a vast and nuanced research program, but all of the analyses I have studied have fallen afoul of the logical considerations of §3.

The basic idea of a three-valued approach is to assign $\phi \to \psi$ to the truth-value assigned to ψ, if ϕ is assigned 1 (true). Otherwise, the conditional is assigned i (indeterminate). To block Material Negation $\neg(\phi \to \psi) \vDash \phi \wedge \neg\psi$ and the even more garish $\phi \to \psi \vDash \phi \wedge \psi$, they must require that a valid argument does not merely preserve truth. It must also guarantee that if the conclusion is false at least one of the premises is false (McDermott 1996: 31). It is far from clear how such a definition should be motivated, but setting this aside, more concrete difficulties emerge. Modus ponens becomes invalid in a way that is even less plausible than the typical 'counterexamples'. $(\phi \to \phi) \to \phi, \phi \to \phi \vDash \phi$ comes out invalid. When the conclusion is false none of the premises have truth-values. These theories also end up requiring profligate meanings for the other sentential connectives (e.g. McDermott 1996: 5). While suppressing Material Negation, they validate a different paradox of material implication IO: $(\phi_1 \wedge \phi_2) \to \psi \vDash (\phi_1 \to \psi) \vee (\phi_2 \to \psi)$. If the premise is true ϕ_1, ϕ_2 and ψ are true, so the conclusion is too. If the conclusion is false ϕ_1 and ϕ_2 are true and ψ false, in which case the premise is false too. A more minor concern is that three-valued accounts invalidate contraposition. When $\neg\phi \to \neg\psi$ is false $\phi \to \psi$ will be undefined.

Huitink (2008: §5.3) attempts to address the failure of contraposition. She too appeals to a version of 'Strawson Validity': $\phi_1 \vDash_S \psi$ iff $\phi_1, \phi_2 \vDash \psi$, where $\vDash$ is classical validity and ϕ_2's truth guarantees that ϕ_1 and ψ have classical truth-values. Since $\phi \to \psi, \neg\psi \vDash \neg\psi \to \neg\phi$, contraposition is Strawson Valid even on a trivalent semantics. Unfortunately, Material Negation is Strawson Valid too, since $\neg(\phi \to \psi), \phi \vDash \phi$. Even more gruesome is the result that $\phi \to \psi \vDash_S \phi \wedge \psi$, since $\phi \to \psi, \phi \vDash \phi \wedge \psi$. So this attempt to bring the logic closer to intuition, recreates more problems than it solves. Even if a technical could be found for these problems, the proposal suffers from a more basic flaw. It is built on the idea that some inferences like contraposition have an implicit premise which is that all of the sentences involved have truth-values (Huitink 2008: 174). But there is no plausibility to the claim that intuitions about the validity of contraposition rely on the implicit premise that the antecedent of the conclusion ($\neg\psi$) is true. Consider the following line of reasoning. Bob might have danced and if he did, Leland danced. So Leland might have danced, but if he didn't, Bob didn't either. Here, contraposition sounds correct despite the fact that the antecedent of the conclusion is explicitly *not* accepted.[27] Jeffrey (1963: 39) validates contraposition by a different route but also at the cost of validating Material Negation. Furthermore, there is a broader problem looming due to the fact that these accounts draw no link between indicative conditionals and epistemic modals. First, they fail to validate an entailment that captures the intuitive point of a negated indicative conditional: $\neg(\phi \to \psi) \vDash \Diamond(\phi \wedge \neg\psi)$; when it is known that $\neg\phi$, the conclusion is false but the premise is undetermined. Second, for the reasons detailed in §3 any account that wishes to explain the gauntlet of counterexamples to various principles needs to appeal to the *possibility* of the antecedent, not the antecedent's truth. But appealing to this has no force when the

[27] An alternative appeal to reasonable inference creates the same problems (Huitink 2008: §5.3.2).

semantics is not modal. Any successful account of indicative conditionals must link it to a state of information which is dynamically influenced in the process of drawing inferences and interpreting conditionals. Without doing so, it will not be possible to account for the new counterexamples to logical principles containing modals discussed in §3.1. A final noteworthy difficulty for trivalent accounts arises from the fact that they offer no satisfactory account of the relationship between indicative and subjunctive conditionals. By contrast, the semantics here could be combined with a semantics for the distinctive morphology of subjunctive conditionals to yield a uniform theory (Starr forthcoming).

5 Conclusion

This paper has identified and steered through a myriad of choice-points in developing a strict analysis of indicative conditionals, and it is time to take stock. I began by arguing for a strict analysis which takes seriously the idea that indicative conditionals involve dynamic transformations of an information state, just as Peirce intimated. This requires a semantics where information states capture the epistemic perspective of an agent. That requires allowing the information state to be inconsistent with the way the world of evaluation is (Reflexivity) without invalidating Modus Ponens. The strict dynamic semantics of Definition 3 achieved this admirably, but broadened the focus of logic. Logic was no longer construed as articulating the laws of truth, but as the laws of information flow. This new perspective was then tested in a minefield few survive: the logic of indicative conditionals. It was argued there that once suitably augmented the dynamic strict analysis fares better than the existing alternatives. Most importantly, it is the only one on offer that correctly diagnoses new counterexamples to a number of inference patterns that are predicted by other accounts to either be semantically valid or pragmatically correct. Finally, I showed how the strict dynamic semantics plus a perfectly general truth definition endows indicative conditionals with context-independent trivalent truth conditions. This allows these truth-conditions to be made available on-demand in the compositional semantics. This was exploited in the solution of a puzzle regarding the interaction of conditionals and *probably*. While it has been known for awhile that a solution based on trivalent truth-conditions would work, it has generally been dismissed on logical grounds – namely those just articulated in §4.2. But since the truth-conditions play no role in the logic of indicative conditionals developed above, this problem dissolved. While there is certainly much left to be explored, I regard these results as powerful reasons to count the dynamic strict analysis among our most successful analyses of indicative conditionals.

A Dynamic Strict-Conditional Logic (DSL)

A.1 Syntax

Definition 15 (DSL Syntax)

(1) $A \in \mathcal{W}\!f\!f$ if $A \in \mathcal{A}t = \{\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{A}_0, \mathsf{A}_1, \ldots\}$
(2) $\neg\phi \in \mathcal{W}\!f\!f$ if $\phi \in \mathcal{W}\!f\!f$
(3) $\Diamond\phi \in \mathcal{W}\!f\!f$ if $\phi \in \mathcal{W}\!f\!f$
(4) $\Box\phi \in \mathcal{W}\!f\!f$ if $\phi \in \mathcal{W}\!f\!f$
(5) $(\phi \wedge \psi) \in \mathcal{W}\!f\!f$ if $\phi, \psi \in \mathcal{W}\!f\!f$
(6) $(\phi \vee \psi) \in \mathcal{W}\!f\!f$ if $\phi, \psi \in \mathcal{W}\!f\!f$
(7) $\phi \to \psi \in \mathcal{W}\!f\!f$ if $\phi, \psi \in \mathcal{W}\!f\!f$

A.2 States and Operations on Them

Definition 16 (Worlds)
$W : \mathcal{A}t \mapsto \{1, 0\}$ where $\mathcal{A}t = \{\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{A}_0, \mathsf{A}_1, \ldots\}$

Definition 17 (Information States) $S = \{s \subseteq W\}$

A.3 Update Semantics

Definition 18 (Update Semantics) $[\cdot] : (\mathcal{W}\!f\!f \times S) \mapsto S$

(1) $s[A] = \{w \in s \mid w(A) = 1\}$
(2) $s[\neg\phi] = s - s[\phi]$
(3) $s[\phi \wedge \psi] = (s[\phi])[\psi]$
(4) $s[\phi \vee \psi] = s[\phi] \cup s[\psi]$
(5) $s[\Diamond\phi] = \{w \in s \mid s[\phi] \neq \varnothing\}$
(6) $s[\Box\phi] = \{w \in s \mid s[\phi] = s\}$

Definition 19 (Presuppositional Dynamic Strict Conditional)

$$s[\phi \to \psi] = \begin{cases} s & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s[\phi] \vDash \psi \\ \varnothing & \text{if } \exists w \in s\colon w \vDash \phi \ \&\ s[\phi] \nvDash \psi \\ \text{Undefined} & \text{otherwise} \end{cases}$$

A.4 Semantic Concepts

Definition 20 (Semantic Concepts)

(1) Support $s \vDash \phi \Leftrightarrow s[\phi] = s$
(2) Truth in w $w \vDash \phi \Leftrightarrow \{w\}[\phi] = \{w\}$
(3) Inconsistency $\nexists s\colon s[\phi][\psi] \neq \varnothing$
(4) Incoherence $\nexists s\colon s \vDash \phi \ \&\ s \vDash \psi$
(5) Propositions $[\![\phi]\!] = \{w \mid w \vDash \phi\}$

Definition 21 (Dynamic Strawsonian Entailment)
$\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall s\colon$ if $s[\phi_1]\cdots[\phi_n][\psi]$ is defined, $s[\phi_1]\cdots[\phi_n] \vDash \psi$

Definition 22 (Strawsonian Logical Truth)
$\vDash \phi \Leftrightarrow \forall s\colon$ if $s[\phi]$ is defined, $s \vDash \psi$

A.5 Logical Validities

A.5.1 Persistence and Preservation

Here I define two properties of modal formulae in DSL and show which modal formulae have which of the properties.

Definition 23 (Persistence) ϕ is persistent iff $s' \vDash \phi$ if $s \vDash \phi$ and $s' \subseteq s$. (I.e. ϕ's support persists after more information comes in.)

Fact 7 In general, $\Diamond\phi$ is not persistent. Take a s containing many worlds but only one ϕ-world w. Then $s \vDash \Diamond\phi$, but $s - \{w\} \subseteq s$ and $s - \{w\} \nvDash \Diamond\phi$.

Fact 8 $\phi \to \psi$ is persistent if its constituents are. Suppose $s \vDash \phi \to \psi$. Then $s[\phi][\psi] = s[\phi]$. If both ϕ and ψ are persistent and $s' \subseteq s$ then $s'[\phi][\psi] = s'[\phi]$, hence $s'[\phi] \vDash \psi$ and so $s' \vDash \phi \to \psi$. So $\phi \to \psi$ is persistent too.

Remark 1 $\Diamond\phi$ is equivalent to $\neg((\phi \vee \neg\phi) \to \neg\phi)$, so there are non-persistent formulae even in the $\Diamond$-free fragment.

Fact 9 $\Box\phi$ is persistent if ϕ is. Suppose $s \vDash \Box\phi$. Then $s \vDash \phi$. If $s' \subseteq s$ and ϕ is persistent, then $s' \vDash \phi$ and hence $s' \vDash \Box\phi$.

Definition 24 (Miserly) ϕ is miserly iff $s'[\phi] \subseteq s[\phi]$ if $s' \subseteq s$ (equivalently: $s' \nvDash \phi$ if $s \nvDash \phi$ and $s' \subseteq s$). I.e. as information improves the failure to support ϕ is preserved.

Fact 10 $\Box\phi$ is not miserly. Consider a s such that $(s - s[\phi]) \neq \varnothing$. Then $s[\Box\phi] = \varnothing$. Let $s' = s[\phi]$. Then $s'[\Box\phi] = s'$. So $s' \subseteq s$ but $s'[\Box\phi] \nsubseteq s[\Box\phi]$.

Fact 11 $\phi \to \psi$ is not miserly. Consider a s such that $(s - s[\phi \wedge \neg\psi]) \neq \varnothing$. Then $s[\phi \to \psi] = \varnothing$. Let $s' = s - s[\phi \wedge \neg\psi]$. Then $s'[\phi \to \psi] = s'$. So $s' \subseteq s$ but $s'[\phi \to \psi] \nsubseteq s[\phi \to \psi]$.

A.5.2 Validities

Remark 2 When proving a validity $\phi \vDash \psi$ below, I will follow Definition 21 and assume that $s[\phi][\psi]$ is defined. My goal will be to show that $s[\phi][\psi] = s[\phi]$.

Fact 12 (Deduction Equivalence) $\phi \vDash \psi \iff \vDash \phi \to \psi$

Proof $\phi \vDash \psi$ holds just in case $\forall s: : s[\phi][\psi] = s[\phi]$. $\vDash \phi \to \psi$ holds just in case $\forall s{:}\, s[\phi \to \psi] = s$. By the semantics of $\to$ this amounts to $s[\phi][\psi] = s[\phi]$. Thus, the equivalence holds.

Fact 13 (Modus Ponens) $\phi \to \psi, \phi \vDash \psi$

Proof Either $s[\phi \to \psi] = s$ or $s[\phi \to \psi] = \varnothing$. In the former case $s[\phi \to \psi][\phi][\psi] = s[\phi \to \psi][\phi]$ is equivalent to $s[\phi][\psi] = s[\phi]$, and it is also guaranteed that $s[\phi] \vDash \psi$. By the last point it follows that $s[\phi][\psi] = s[\phi]$ and hence by the equivalence that $s[\phi \to \psi][\phi][\psi] = s[\phi \to \psi][\phi]$. In the latter case $s[\phi \to \psi][\phi][\psi] = \varnothing = s[\phi \to \psi][\phi]$.

Fact 14 (Identity) $\vDash \phi \to \phi$

Proof $s[\phi \to \phi] = \{w \in s \mid s[\phi] \vDash \phi\} = \{w \in s \mid s[\phi][\phi] = s[\phi]\}$. So the validity comes down to the Idempotence of ϕ. Suppose ϕ is not Idempotent. Since update is eliminative ($s[\phi] \subseteq s$) Idempotency must fail because of the other direction, in which case: $s[\phi] \nsubseteq s[\phi][\phi]$. Take an arbitrary $w \in s$ and suppose that $\{w\}[\phi] = \{w\}$. Then $\{w\}[\phi][\phi] = \{w\}$. Note that if $w \in s \cap [\![\phi \wedge \phi]\!]$ then $w \in s[\phi][\phi]$ (the converse is not true!). So $w \in s[\phi][\phi]$. But then we've shown that $s[\phi] \subseteq s[\phi][\phi]$, which contradicts our assumption of non-Idempotence. Then it follows from non-Idempotence that $\nexists w \in s{:}\, \{w\}[\phi] = \{w\}$. In that case the presupposition of the conditional is not met when the antecedent is non-Idempotent, so such instances do not bear on Identity's validity. Thus, the validity of the pattern for Idempotent cases is sufficient to show the validity in general.

Remark 3 Here's a non-Idempotent case: $(\Diamond \mathsf{A} \wedge \neg \mathsf{A}) \to (\Diamond \mathsf{A} \wedge \neg \mathsf{A})$. This will amount to testing that $s[\Diamond \mathsf{A}][\neg \mathsf{A}] \vDash \Diamond \mathsf{A} \wedge \neg \mathsf{A}$. But this test will fail, since after taking in $\neg \mathsf{A}$ the information no longer supports the first conjunct $\Diamond \mathsf{A}$. However, $\nexists w \in s{:}\, \{w\}[\Diamond \mathsf{A} \wedge \neg \mathsf{A}]$, since no world makes that formula true. The insight behind the proof is that any failure of Idemopotence is going to be like this.

Fact 15 (Import-Export) $\phi_1 \to (\phi_2 \to \psi) \dashv\vDash \phi_1 \wedge \phi_2 \to \psi$

Proof
$$\begin{aligned} s[\phi_1 \to (\phi_2 \to \psi)] &= \{w \in s \mid s[\phi_1] \vDash \phi_2 \to \psi\} \\ &= \{w \in s \mid s[\phi_1][\phi_2 \to \psi] = s[\phi_1]\} \\ &= \{w \in s \mid \{w' \in s[\phi_1] \mid s[\phi_1][\phi_2] \vDash \psi\} = s[\phi_1]\} \\ &= \{w \in s \mid s[\phi_1][\phi_2] \vDash \psi\} \\ &= \{w \in s \mid s[\phi_1 \wedge \phi_2] \vDash \psi\} \\ &= s[\phi_1 \wedge \phi_2 \to \psi] \end{aligned}$$

Fact 16 (Might) $\phi \to \Diamond \psi \dashv\vDash \Diamond(\phi \wedge \psi)$

Proof
$$\begin{aligned} s[\phi \to \Diamond \psi] &= \{w \in s \mid s[\phi] \vDash \Diamond \psi\} \\ &= \{w \in s \mid s[\phi][\Diamond \psi] = s[\phi]\} \\ &= \{w \in s \mid s[\phi][\psi] \neq \varnothing\} \\ &= \{w \in s \mid s[\phi \wedge \psi] \neq \varnothing\} \\ &= s[\Diamond(\phi \wedge \psi)] \end{aligned}$$

Fact 17 (Must) $\Box \phi \to \psi \dashv\vDash \phi \to \psi \dashv\vDash \phi \to \Box \psi$

Proof
$$\begin{aligned} s[\Box(\phi \to \psi)] &= \{w \in s \mid s[\phi \to \psi] = s\} \\ &= \{w \in s \mid s[\phi] \vDash \psi\} \\ &= s[\phi \to \psi] \\ &= \{w \in s \mid s[\phi][\psi] = s[\phi]\} \\ &= \{w \in s \mid s[\phi][\Box \psi] = s[\phi]\} \\ &= \{w \in s \mid s[\phi] \vDash \Box \psi\} \\ &= s[\phi \to \Box \psi] \end{aligned}$$

Fact 18 (Modus Tollens) For miserly ψ, $\phi \to \psi, \neg\psi \vDash \neg\phi$

Proof Suppose ψ is miserly. If the update with the conclusion is defined, the test imposed by the premise must be successful and so $s[\phi][\psi] = s[\phi]$. To show that the inference is valid, we must show that $s[\neg\psi] \vDash \neg\phi$. This amounts to $s[\neg\psi][\neg\phi] = s[\neg\psi]$. Since update is eliminative, $s[\neg\psi][\neg\phi] \subseteq s[\neg\psi]$. Hence we must show that $s[\neg\psi] \subseteq s[\neg\psi][\neg\phi]$. This simplifies to $(s - s[\psi]) - (s - s[\psi])[\phi] \subseteq s - s[\psi]$. For reductio, suppose w is not in the set named on the left, but is in the set named on the right. In virtue of the former fact $w \in (s - s[\psi])[\phi]$. If ϕ is miserly, it follows that $w \in s[\phi]$, since $s - s[\psi] \subseteq s$. Then it follows that $w \in s[\phi][\psi]$. Since ψ is miserly and $s[\phi] \subseteq s$, $w \in s[\psi]$. This is a contradiction since $w \in s - s[\psi]$. If ϕ isn't miserly, the only way for $(s - s[\psi])[\phi] \subseteq s[\phi]$ to fail is for $s[\phi] = \varnothing$ and $(s - s[\psi])[\phi] \neq \varnothing$. But this cannot occur since the premise presupposes that $s[\phi] \neq \varnothing$ and the presupposition is assumed to be met. Hence ϕ need not be miserly.

Remark 4 $\mathsf{A} \to \Box\mathsf{B}, \neg\Box\mathsf{B} \nvDash \neg\mathsf{A}$. Suppose we know that either (i) a and b are squares or (ii) a is a circle and b is a square or (iii) both a and b are circles. Then, if a is a square, b must be a square. Also, it's not the case that b must be a square. But it does not follow that a is not a circle.

Remark 5 $\mathsf{A} \to (\mathsf{B} \to \mathsf{C}), \neg(\mathsf{B} \to \mathsf{C}) \nvDash \neg\mathsf{A}$. Suppose we know that either (i) a, b and c are squares or (ii) b is a square, but a and s are circles. So, if a is a square, then if b is a square s is too. Also, it's false that if b is a square, s is a square; after all b could be a square while s is a circle. Yet it does not follow that a is not a square.

Fact 19 (Contraposition) For miserly ψ, $\phi \to \psi \vDash \neg\psi \to \neg\phi$

Proof Suppose ψ is miserly. If the update with the conclusion is defined, the test imposed by the premise must be successful and so $s[\phi][\psi] = s[\phi]$. To show that the inference is valid, we must show that $s[\neg\psi] \vDash \neg\phi$. This has already been shown to hold under these conditions in the proof of Fact 18.

Remark 6 Consider $\mathsf{A} \to \Box\mathsf{B}$. Contraposition would allow us to infer $\neg\Box\mathsf{B} \to \neg\mathsf{A}$, i.e. $\Diamond\neg\mathsf{B} \to \neg\mathsf{A}$.

Fact 20 (AS) For persistent ψ, $\phi_1 \to \psi \vDash (\phi_1 \wedge \phi_2) \to \psi$

Proof If $s[\phi_1 \to \psi][(\phi_1 \wedge \phi_2) \to \psi]$ is defined, $s[\phi_1 \to \psi] = s$ and so $s[\phi_1] \vDash \psi$. $s[\phi_1][\phi_2] \subseteq s[\phi_1]$ and since ψ is persistent, $s[\phi_1][\phi_2] \vDash \psi$. Thus, $s[(\phi_1 \wedge \phi_2) \to \psi] = s$ and hence $s[\phi_1 \to \psi][(\phi_1 \wedge \phi_2) \to \psi] = s[\phi_1 \to \psi]$.

Remark 7 $\mathsf{A} \to \Diamond\mathsf{B} \nvDash \mathsf{A} \wedge \neg\mathsf{B} \to \Diamond\mathsf{A}$. Let s contain one $\mathsf{A} \wedge \mathsf{B}$-world and one $\mathsf{A} \wedge \neg\mathsf{B}$-world. Then $s[\mathsf{A} \to \Diamond\mathsf{B}] = s$. But $s \nvDash \mathsf{A} \wedge \neg\mathsf{B} \to \Diamond\mathsf{B}$, since $s[\mathsf{A}][\neg\mathsf{B}] \nvDash \Diamond\mathsf{B}$. After all, $s[\mathsf{A}][\neg\mathsf{B}][\Diamond\mathsf{B}] = \varnothing \neq s$.

Fact 21 (SDA) For persistent ψ, $(\phi_1 \vee \phi_2) \to \psi \vDash (\phi_1 \to \psi) \wedge (\phi_2 \to \psi)$

Proof The premise tests that $s[\phi_1] \cup s[\phi_2] \vDash \psi$. The conclusion presupposes that $s[\phi_1] \neq \varnothing$ and $s[\phi_2] \neq \varnothing$, and tests that $s[\phi_1] \vDash \psi$ and $s[\phi_2] \vDash \psi$. Since $s[\phi_1] \subseteq (s[\phi_1] \cup s[\phi_2])$ and $s[\phi_2] \subseteq (s[\phi_1] \cup s[\phi_2])$, this test must be successful when ψ is persistent but may not be successful when ψ isn't persistent.

Remark 8 $(\mathsf{A} \vee \neg\mathsf{A}) \to \Diamond\mathsf{A} \not\models (\mathsf{A} \to \Diamond\mathsf{A}) \wedge (\neg\mathsf{A} \to \Diamond\mathsf{A})$. If there are both A and $\neg\mathsf{A}$ worlds in s all presuppositions will be met and the premise will successfully test s. The second conjunct of the conclusion won't.

Fact 22 (Transitivity) For persistent ψ, $\phi_1 \to \phi_2, \phi_2 \to \psi \models \phi_1 \to \psi$

Proof If $s[\phi_1 \to \phi_2][\phi_2 \to \psi][\phi_1 \to \psi]$ is defined:

(41) $s[\phi_1 \to \phi_2][\phi_2 \to \psi] = s$.

Thus, we must show that $s[\phi_1 \to \psi] = s$. This amounts to showing that $s[\phi_1] \models \psi$, i.e. $s[\phi_1][\psi] = s[\phi_1]$. Fact (41) requires $s[\phi_2] \models \psi$, hence:

(42) $s[\phi_2][\psi] = s[\phi_2]$

Assume ϕ_2 is miserly. Then, since $s[\phi_1] \subseteq s$:

(43) $s[\phi_1][\phi_2] \subseteq s[\phi_2]$

Together with ψ's persistence (42) and (43) entail that $s[\phi_1][\phi_2][\psi] = s[\phi_1][\phi_2]$. But then $s[\phi_1][\psi] = s[\phi_1]$, since $s[\phi_1][\phi_2] = s[\phi_1]$. After all, Fact (41) requires that $s[\phi_1] \models \phi_2$ and hence $s[\phi_1][\phi_2] = s[\phi_1]$. If ϕ_2 is not miserly the only way for $s[\phi_1][\phi_2] \subseteq s[\phi_2]$ to fail is for $s[\phi_2] = \varnothing$ and $s[\phi_1][\phi_2] \neq \varnothing$. But that can't happen since $\phi_2 \to \psi$ presupposes that $s[\phi_2] \neq \varnothing$. Hence the argument above goes through without the assumption that ϕ_2 is miserly.

Remark 9 $\neg\mathsf{A} \to \mathsf{B}, \mathsf{B} \to \Diamond\mathsf{A} \not\models \neg\mathsf{A} \to \Diamond\mathsf{A}$. Let $s = \{w_0, w_1, w_2\}$, where w_0 is a $\mathsf{A} \wedge \mathsf{B}$-world, w_1 is a $\neg\mathsf{A} \wedge \mathsf{B}$-world and w_2 is a $\mathsf{A} \wedge \neg\mathsf{B}$-world. The first premise successfully tests s, since the only $\neg\mathsf{A}$- world in s is a B-world, namely w_1. The second premise is also successful since one of the B-worlds is a A-world, namely w_0. But the conclusion fails: among the $\neg\mathsf{A}$-worlds in s there are no A-worlds!

Fact 23 (True/Necessary Consequent) For persistent ψ, $\psi \models \phi \to \psi$.

Proof $s[\psi][\phi] \models \psi$ holds if ψ is persistent, which it is by assumption. Thus the pattern is valid.

Remark 10 Consider whether $\Diamond\mathsf{A} \models \neg\mathsf{A} \to \Diamond\mathsf{A}$. The premise requires $\Diamond\mathsf{A}$ requires some $w_1 \in s : w_1(\mathsf{A}) = 1$, and the antecedent $\neg\mathsf{A}$ requires some $w_0 \in s : w_0(\mathsf{A}) = 0$. So let $s = \{w_1, w_0\}$. Since $s[\Diamond\mathsf{A}] = s$, $s[\Diamond\mathsf{A}][\neg\mathsf{A}] = s[\neg\mathsf{A}]$, so the validity of TC would require $s[\neg\mathsf{A}][\Diamond\mathsf{A}] = s[\neg\mathsf{A}]$. But $s[\neg\mathsf{A}] = \{w_0\}$ and $\{w_0\}[\Diamond\mathsf{A}] = \varnothing$.

Fact 24 (Antecedent Persistence) For persistent ϕ, $\models \phi \to (\psi \to \phi)$.

Proof AS follows immediately from Fact 23 and 12.

Remark 11 Consider whether $\models \Diamond\mathsf{A} \to (\neg\mathsf{A} \to \Diamond\mathsf{A})$. This will fail for exactly the same reason the instance discussed in Remark 10 fails.

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