Expressing Choices

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Outline

Possible Worlds and Information

In Inquiry and Communication (Stalnaker 1984)

- Informational contents (*propositions*) are sets of possible worlds
  - They distinguish ways world might be (worlds in the set) from ways it isn't (worlds excluded from set)
- **Rationality**: propositions are the objects of attitudes
- **Communication**: contents 'transmitted' by assertions

State of Information (s)

As communication and inquiry unfold, a body of information accumulates. Think of this information as what the agents are mutually taking for granted. Call the set of worlds embodying this information s, short for the *state of information*. (Stalnaker 1978; Lewis 1979)

Gaining Information

And Eliminating Possibilities

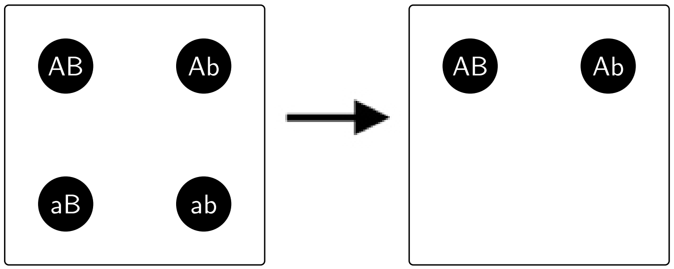


Figure: Accepting the information that A

- Inquiry progresses by using information to reduce uncertainty, i.e. eliminate worlds.
- $\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\} \Rightarrow \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$

The Role of Semantics

In the Modal Orthodoxy

Classical Picture

1. **Semantics**: pair sentences w/propositions
   - $[\![\phi]\!]$ is a set of worlds
2. **Pragmatics**: rules for rational agents
   - When presented with information, rational agents use it to eliminate possibilities (decrease uncertainty)

Modal Orthodoxy

Representational Semantics

Orthodox Possible Worlds Semantics

1. $[\![\mathsf{A}]\!] = \{w \mid w(\mathsf{A}) = 1\}$
2. $[\![\neg\phi]\!] = W - [\![\phi]\!]$
3. $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$
4. $[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$
5. $[\![\Diamond\phi]\!] = \{w \mid \exists w' \colon \in R(w, w') \;\&\; w' \in [\![\phi]\!]\}$
   - $R(w, w')$: w' is 'accessible' from w

Classical Truth and Consequence

Truth $w \vDash \phi \iff w \in [\![\phi]\!]$

Consequence $\phi \vDash \psi \iff [\![\phi]\!] \subseteq [\![\psi]\!]$

Two Consequences of the Orthodoxy

Possibility and Disjunction

Fact 1: $\Diamond \mathsf{A} \vee \Diamond \mathsf{B} \nvDash \Diamond \mathsf{A}$ and $\Diamond(\mathsf{A} \vee \mathsf{B}) \nvDash \Diamond \mathsf{A}$

1. First would require:
   - $[\![\Diamond\mathsf{A}]\!] \cup [\![\Diamond\mathsf{B}]\!] \subseteq [\![\Diamond\mathsf{A}]\!]$
   - But this only holds when $[\![\Diamond\mathsf{B}]\!] = \varnothing$
2. Second would require:
   - $[\![\mathsf{A} \vee \mathsf{B}]\!] \subseteq [\![\mathsf{A}]\!]$
   - Would hold only when $[\![\mathsf{B}]\!] = \varnothing$

- Relatedly: $\neg\Diamond(\mathsf{A} \vee \mathsf{B}) \vDash \neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}$

Two Consequences of the Orthodoxy

Explaining Why $\Diamond\mathsf{A}$ and $\neg\Diamond\mathsf{A}$ are Inconsistent

Fact 2: $[\![\Diamond\mathsf{A}]\!] \cap [\![\neg\Diamond\mathsf{A}]\!] = \varnothing$

- Fact taken to explain why asserting/believing both is dysfunctional/irrational
- **Assumption 1**: function of assertion/belief is to represent how the world is
- **Assumption 2**: $[\![\cdot]\!]$ is the representation relation
- **Explanation**: no world can be both how $\Diamond\mathsf{A}$ and $\neg\Diamond\mathsf{A}$ represent the world as being, so it is dysfunctional to assert/believe both

- Do all modal claims represent 'modal reality'?

Free Choice Permission

Data from Natural Language

(1) a. You may vote for Anderson or Brady
b. You may vote for Anderson
c. You may vote for Brady

Narrow Free Choice Permission (NFC)

1. May $(A \vee B) \Longrightarrow$ May A
2. May $(A \vee B) \Longrightarrow$ May B
   - '$\Longrightarrow$': shorthand for 'implication', neutral between semantic consequence and pragmatic implicature

(von Wright 1968: 4-5, Kamp 1973)

Wide Free Choice Permission

Data from Natural Language

(2) a. You may vote for Anderson or you may vote for Brady
b. You may vote for Anderson
c. You may vote for Brady

Wide Free Choice Permission (WFC)

1. May A $\vee$ May B $\Longrightarrow$ May A
2. May A $\vee$ May B $\Longrightarrow$ May B
   - '$\Longrightarrow$': shorthand for 'implication', neutral between semantic consequence and pragmatic implicature

(Guerts 2005; Simons 2005)

Free Choice and the Modal Orthodoxy

Intermediate Conclusion

- **Recall Fact 1**: neither NFC nor WFC are entailments on orthodox approach
- Zimmermann (2000): new semantics for modal sentences containing *or*
  - And predicts NFC as an implicature
- Guerts (2005), Simons (2005): new semantics for *or*, combined w/roughly orthodox modal semantics
  - Predicts NFC and WFC as entailments
  - Predicts May $(A \vee B)$ is equiv. to May A $\wedge$ May B
  - Important advantages over Zimmermann (2000)
- Problem Solved?

Dual Prohibition

More Data

(3) a. You may not vote for Anderson or Brady
b. You may not vote for Anderson
c. You may not vote for Brady

Dual Prohibition (DP)

1. $\neg$May $(A \vee B) \Longrightarrow \neg$May A
2. $\neg$May $(A \vee B) \Longrightarrow \neg$May B
   - '$\Longrightarrow$': shorthand for 'implication', neutral between semantic consequence and pragmatic implicature

(Alonso-Ovalle 2006; Fox 2007)

Dual Prohibition

The Tension between Free Choice and Dual Prohibition

- DP is predicted by orthodox semantics
  - Seems to require that semantics!
- But predicting WFC and NFC required a slightly different orthodoxy (Guerts 2005; Simons 2005)
  - $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})$ as equiv. to $\mathsf{May}\,\mathsf{A} \wedge \mathsf{May}\,\mathsf{B}$
  - In which case $\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})$ only gives you $\neg\mathsf{May}\,\mathsf{A} \vee \neg\mathsf{May}\,\mathsf{B}$
- Birthed new attempts to treat NFC as implicatures
  - Combined radically new way of deriving implicatures (Fox 2007; Franke 2009; van Rooij 2010)
- And radically non-orthodox semantics (Barker 2010)

Resource Sensitivity

Permission as Partial, Discrete

(4) a. You may vote for Anderson or Brady
b. # You may vote for both Anderson and Brady
c. # You may not vote for both Anderson and Brady

(5) a. You may vote for Anderson or Brady
I did vote for Anderson
I may vote for Brady

Resource Sensitivity (RS)

b.
1. $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}) \not\Rightarrow \mathsf{May}\,(\mathsf{A} \wedge \mathsf{B}), \neg\mathsf{May}\,(\mathsf{A} \wedge \mathsf{B})$
   - Not satisfied by some implicature approaches (As observed by Barker 2010)
2. $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}), \mathsf{A} \not\Rightarrow \mathsf{May}\,\mathsf{B}$

Out of the Rabbit Hole

Theoretical Wishlist

Wishlist

1. Predict (Narrow/Wide) Free Choice Implications
2. Predict Dual Prohibition Implications
3. Predict Resource Sensitivity Implications

Hunch

- Tension between 1 and 2 product of purely representational semantics for modals and connectives
- 3 suggests that deontic modals incrementally build and remove partial permissions

Deontic Discourse

And Motivation

Deontic Discourse

And Motivation

Deontic Discourse

How Does a Representational Modal Semantics Motivate?

Way Out?

From Accessibility to Preference

- Perhaps Modal Orthodoxy can be adapted
- Replace R with a preference relation $\succ$
  - $w_1 \succ_w w_2$: w_1 is strictly preferable to w_2 in w
- Why?
  - Preferences motivate choice
  - So if deontic modals constrain preferences, they constrain choices

How Preference Constrains Choice (One Possibility)

$\mathit{Choice}(\succ)$ is the set of w' s.t. there is no $w'' \succ_w w'$

- Non-dominance conception of rational choice

Adapting Standard Approach

Deontic Modality and Preference

Descriptivist Preference Semantics (Lewis, Hansson)

$[\![\mathsf{Must}\,\phi]\!]_{\succ} = \{w \mid \forall w_1, w_2 \colon w_2 \succ_w w_1 \text{ if } w_2 \in [\![\phi]\!]_{\succ} \;\&\; w_1 \notin [\![\phi]\!]_{\succ}\}$

- Must ϕ is true in w just in case every ϕ-world is (strictly) preferred in w to every $\neg\phi$-world

- Deontic propositions are about preferences
- Preferences are a feature of 'the world'
- **Problem**: It's not the world at large, but agents in the world who have preferences

Relativizing Orthodox Semantics

Deontic Modality and Preference

Subjectivist Preference Semantics

$[\![\mathsf{Must}\,\phi]\!]_{>A} =$

$\{w \mid \forall w_1, w_2 \colon w_2 >_{A(w)} w_1 \text{ if } w_2 \in [\![\phi]\!]_{>A} \;\&\; w_1 \notin [\![\phi]\!]_{>A}\}$

- $\mathsf{Must}\,\phi$ is true in w just in case every ϕ-world is (strictly) preferred by A in w to every $\neg\phi$-world
- Two variants: A = speaker; A = assessor

- Deontic propositions are about agents' preferences
- **Three Obstacles**:
  1. Makes disagreement difficult to explain (Moore 1912)
  2. Unclear how S informing H about S's preferences constrains H's preferences
  3. Unclear how S can inform H about H's preferences

The Attraction of Expressivism

Deontic Claims Don't Describe Preferences, They Express Them

Expressivist Theses

1. **Communication**: "To express a state of mind is not to say that one is in it" (Gibbard 1986: 473).
2. **Explanation**: "The semantic properties of sentences are to be explained, fundamentally, in terms of properties of the attitudes conventionally expressed by utterances of those sentences" (Silk 2014: §1).
3. **Non-representation**: The states of mind expressed by sentences are non-representational, and, more specifically, motivational.

- Recall Fact 2: expressivist can't adopt *that* explanation of inconsistency

The Catch of Expressivism

What *is* Expressing a State of Mind without Describing It?

The Negation Problem

What states of mind do $\mathsf{Must\,A}$, $\mathsf{Must\,\neg A}$, and $\neg\mathsf{Must\,A}$ express such that jointly asserting/believing $\mathsf{Must\,A}$ and either $\mathsf{Must\,\neg A}$ or $\neg\mathsf{Must\,A}$ is dysfunctional?

- Gibbard (2003: 71-5) tries to live without a positive answer to this question
  - Consensus: you can't (Dreier 2006, 2009; Silk 2014)
- Silk (2014) and Yalcin (2012) try to adapt truth-conditional semantics to the task
- These attempts either lapse back in to descriptivism or fail to solve the problem fully (Starr 2016)

Alternative Model of Expressing Preferences

Building Partial Preference Relations

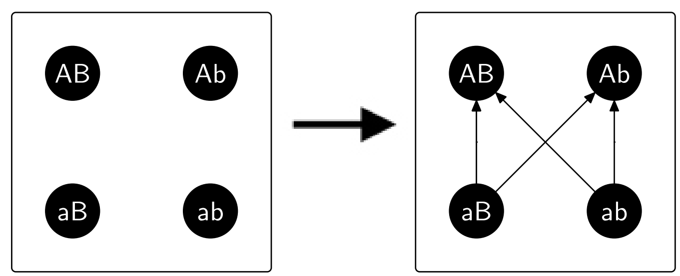


Figure: Preferences Expressed by $\mathsf{Must\,A}$

- $\langle s_0, \varnothing\rangle \Rightarrow$
  $\langle s_0, \{\langle w_{\mathsf{AB}}, w_{\mathsf{aB}}\rangle, \langle w_{\mathsf{AB}}, w_{\mathsf{ab}}\rangle, \langle w_{\mathsf{Ab}}, w_{\mathsf{aB}}\rangle, \langle w_{\mathsf{Ab}}, w_{\mathsf{ab}}\rangle\}\rangle$

Alternative Model of Expressing Preferences

Building Partial Preference Relations

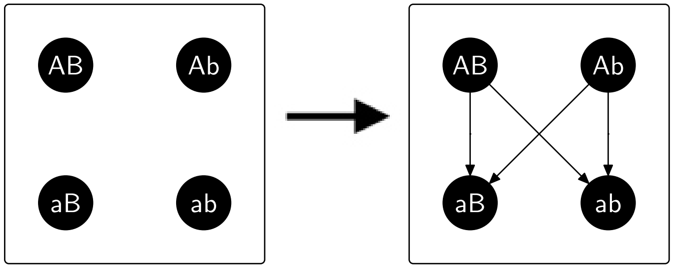
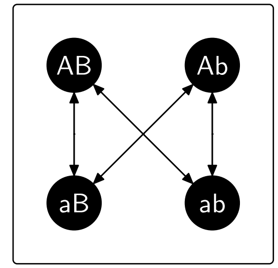


Figure: Preferences Expressed by Must ¬A

Alternative Model of Expressing Preferences

Explaining One Inconsistency (Dreier 2006; Starr 2013; Silk 2014)



$\succ_2$

Figure: Preferences Expressed by Must ¬A and Must ¬A

- Negation problem solved:
  1. Function of deontics is to motivate choice
  2. $Choice(s^{\succ_2}) = \varnothing$, i.e. no alternative can be chosen
  3. So dysfunctional to assert/believe

What It's Like

Alternative Model of Expressing Preferences

External Negation

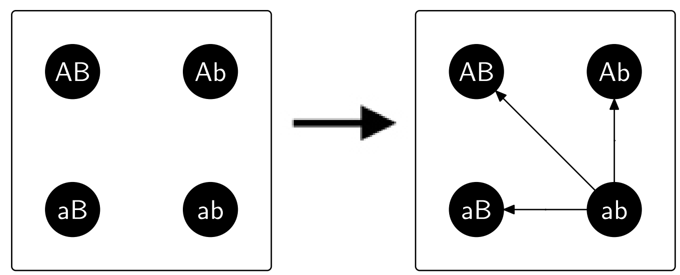
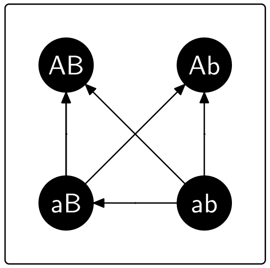


Figure: Preferences Expressed by ¬Must A?

- What semantics for ¬ would deliver this?
- Not the orthodox one! (Frege 1923)

Alternative Model of Expressing Preferences

The Other Inconsistency, Not Explained



$\succ_3$

Figure: Preferences Expressed by Must A and ¬Must A

- Same explanation of inconsistency doesn't work!
- $Choice(s^{\succ_3}) = \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$

Basic Dynamic Semantics

Just Information (Veltman 1996)

Orthodox Picture

- Sentences represent by refer to regions of logical space
- Interpreters use utterances of them to shift to region of logical space within region referred to

Dynamic Semantics (Purely Informational Version)

- Sentences: recipes for moving around logical space
- Atomics: zoom in on a particular region
- Conjunction: apply each recipe in turn
- Disjunction: apply recipes separately; 'merge' results
- Negation: remove region scope would zoom to

The Dynamic Picture

In More Detail

The Basic Idea

Assign each ϕ a function $[\phi]$ encoding how it changes s: $s[\phi] = s'$ (I.e.: $[\phi](s) = s'$)

Dynamic Informational Semantics (Veltman 1996)

1. $s[\mathsf{A}] = \{w \in s \mid w(\mathsf{A}) = 1\}$
2. $s[\neg\phi] = s - s[\phi]$
3. $s[\phi \wedge \psi] = (s[\phi])[\psi]$
4. $s[\phi \vee \psi] = s[\phi] \cup s[\psi]$

A New Dynamic Picture

A Model of Competing Information and Preferences (Starr 2016)

States S

S is a set of substates.

Substates $s^{\succsim}$

A substate $s^{\succsim}$ is a triple consisting of:

1. s an information state, set of worlds
2. $\succ$ a preference ordering on worlds
3. $\sim$ an indifference ordering on worlds

Notation: any set-theoretic operations applied to $s^{\succsim}$ are really applied to s, e.g. $s_0^{\succsim} \cap s_1^{\succsim} := (s_0 \cap s_1)^{\succsim}$

A New Dynamic Picture

The Connective Semantics

Dynamic Connective Semantics (Starr 2016)

1. $S[\mathsf{A}]$: eliminate ¬A-worlds from each substate
2. $S[\neg\phi]$: for each substate,
   a. Eliminate worlds that would survive update w/ϕ
   b. Remove preferences ϕ would add to empty ordering
3. $S[\phi \wedge \psi] = (S[\phi])[\psi]$
4. $S[\phi \vee \psi] = S[\phi] \cup S[\psi]$

- Disjunction will create substates for each disjunct

A New Dynamic Picture

Deontic Semantics for *May*

May

$S[\mathsf{May}\,\phi]$: for each substate $s_i^{\succsim_j}$ in S

- Take each $s_l^{\succsim_k}$ in $\{s_i^{\succsim_j}\}[\phi]$, and test whether the Choice worlds in $s_i^{\succsim_j}$ are consistent with s_l
- If passed, take each s_l and create a substate as follows and add it to S
  - Let $s = \bigcup(\{s_i \mid s_i^{\succsim_j} \in S\})$ be the information and $\succ_{s_l}$ an ordering with preferences only for each s_l world over each $s - s_l$ world
- If failed, return state $\{\varnothing^{\succ_{s_l}}\}$

A Simple Case

Updating with May A

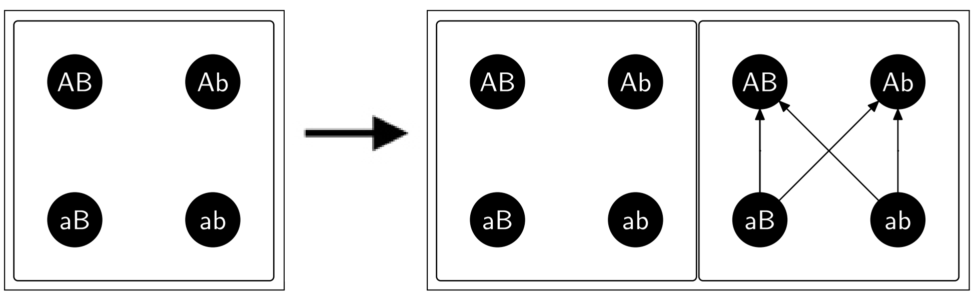


Figure: $\{s_0^{\varnothing}\}[\mathsf{May\ A}]$

- $\{s_0^{\varnothing}\}[\mathsf{A}] = \{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}^{\varnothing}\}$ and $Choice(s_0^{\varnothing}) = s_0$; test ✓
- Add a substate w/info s_0 and a preference only for those A-worlds over rest from s_0

A More Complex Case

Updating with May (A ∨ B)

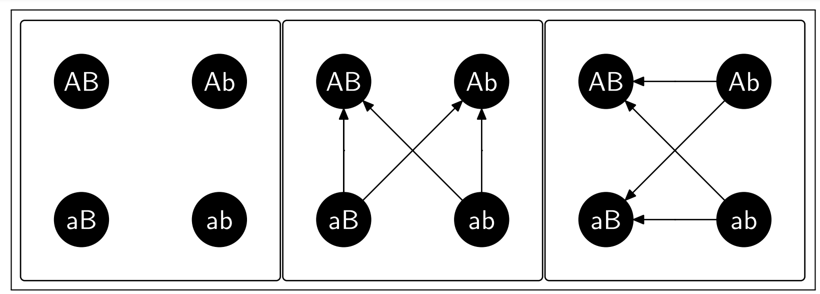


Figure: $\{s_0^{\varnothing}\}[\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})]$

- $\{s_0^{\varnothing}\}[\mathsf{A} \vee \mathsf{B}] = \{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}^{\varnothing}, \{w_{\mathsf{AB}}, w_{\mathsf{aB}}\}^{\varnothing}\}$; tests ✓
- From first one, create new substate with preference for A-worlds and info s_0; same for second one and B-worlds
- Add each to $\{s_0^{\varnothing}\}$

Another Case

Updating with May A ∨ May B

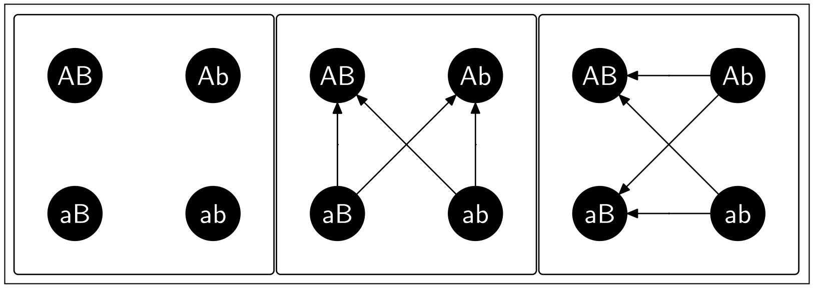


Figure: $\{s_0^{\varnothing}\}[\mathsf{May\,A} \vee \mathsf{May\,B}]$

- Just $\{s_0^{\varnothing}\}[\mathsf{May\,A}] \cup \{s_0^{\varnothing}\}[\mathsf{May\,B}]$

Another Case

Updating with ¬May A

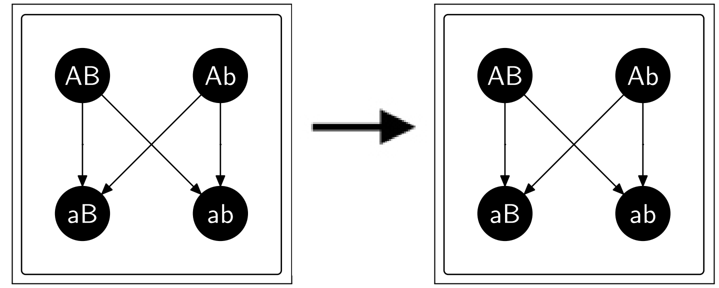


Figure: $\{s_0^{\gtrsim_{\neg A}}\}[\neg\mathsf{May\,A}]$

- Removes worlds that would survive update w/May A
  - None would survive since test fails
- Removes any input preferences May A would add to empty ordering; also idles, no A-worlds preferred in $\gtrsim_{\neg A}$

Another Case

Updating with ¬May (A ∨ B)

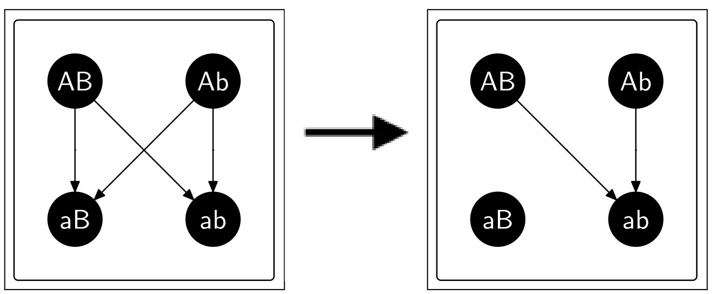


Figure: $\{s_0^{\gtrsim_a}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})]$

- Removes worlds that would survive update w/May (A ∨ B), but none would
  - Test fails on A-worlds
- Removes any input preferences May (A ∨ B) would add to empty ordering; removes B-worlds preferred in $\gtrsim_a$

Updating with ¬May (A ∨ B)

What Kind of State Does it Fit?

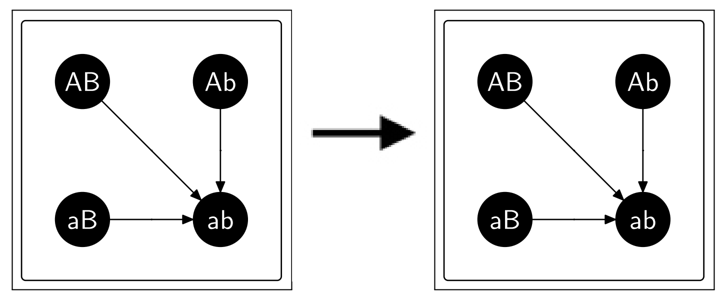


Figure: $\{s_0^{\gtrsim_{ab}}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})]$

- Removes worlds that would survive update w/May (A ∨ B), but none would since both tests fail
- Removes any input preferences May (A ∨ B) would add to empty ordering; but there are none

Towards a Logic

Two Kinds of Support

Informational Support

$S \vDash \phi \iff i_S = i_{S[\phi]}$

- $i_S = \bigcup\{s \mid \exists{\succsim}\colon s^{\succsim} \in S\}$

Preferential Support

$S \Vdash \phi \iff \mathit{Prefs}_S = \mathit{Prefs}_{S[\phi]}$

- $\mathit{Prefs}_S = \{{\succsim} \mid \exists s \neq \varnothing\colon s^{\succsim} \in S\}$

Towards a Logic

Two Kinds of Consequence

Informational Consequence

$\phi_1, \ldots, \phi_n \vDash \psi \iff \forall S\colon S[\phi_1]\cdots[\phi_n] \vDash \psi$

Preferential Consequence

$\phi_1, \ldots, \phi_n \Vdash \psi \iff \forall S\colon S[\phi_1]\cdots[\phi_n] \Vdash \psi$

- More simply: $\phi \Vdash \psi \iff \forall S\colon S[\phi] = S[\phi][\psi]$

Dual Prohibition is Valid

Updating with ¬May (A ∨ B) Preferentially Supports ¬May A and ¬May B

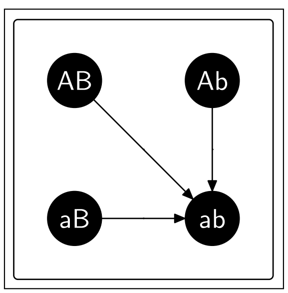


Figure: $\{s_0^{\succsim_{ab}}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})]$

- $\{s_0^{\succsim_{ab}}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})] = \{s_0^{\succsim_{ab}}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})][\neg\mathsf{May}\,\mathsf{A}]$
- $\{s_0^{\succsim_{ab}}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})] = \{s_0^{\succsim_{ab}}\}[\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})][\neg\mathsf{May}\,\mathsf{B}]$

Free Choice is Valid

Updating with May (A ∨ B) or May A ∨ May B...

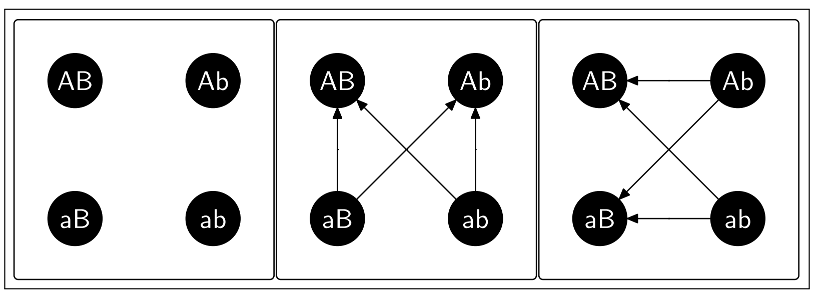


Figure: $\{s_0^{\varnothing}\}[\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})]$

- $\{s_0^{\varnothing}\}[\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})] = \{s_0^{\varnothing}\}[\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})][\mathsf{May}\,\mathsf{A}]$
- And $\mathsf{May}\,\mathsf{A} \vee \mathsf{May}\,\mathsf{B}$ was the same as $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})$
- Both NFC and WFC valid!

Dynamic Expressive Deontic Logic
Interesting...

The Logic
1. (Narrow/Wide) Free Choice Valid
2. Dual Prohibition Valid
3. Resource Sensitivity Valid (not discussed)

Wishlist
1. Predict (Narrow/Wide) Free Choice Implications
2. Predict Dual Prohibition Implications
3. Predict Resource Sensitivity Implications

Explaining Inconsistency
From an Expressivist Perspective

Informational Consistency
$\phi_1, \ldots, \phi_n$ are informationally consistent
$\iff \exists S\colon i_S \neq \varnothing \;\&\; S \vDash \phi_1, \ldots, S \vDash \phi_n$

Preferential Consistency
$\phi_1, \ldots, \phi_n$ are preferentially consistent
$\iff \exists S\colon \mathit{Ch}(S) \neq \varnothing \;\&\; S \Vdash \phi_1, \ldots, S \Vdash \phi_n$

- Where $\mathit{Ch}(S) = \bigcup\{\mathit{Choice}(s, \succsim) \mid s^{\succsim} \in S\}$
- Recall: if $\mathit{Choice}(s, \succsim) = \varnothing$ then $\succsim$ is dysfunctional, i.e. fails to motivate a choice.
  - E.g. if $\succsim$ is cyclic over s, $\mathit{Choice}(s, \succsim) = \varnothing$

Explaining Inconsistency
Preferential Inconsistency (Starr 2016)

- $\mathsf{Must}\,\phi$ and $\mathsf{Must}\,\neg\phi$ are preferentially inconsistent
  - Only irrational states support them, i.e. ones with cyclic preferences
- But $\mathsf{Must}\,\phi$ and $\neg\mathsf{Must}\,\phi$ are preferentially inconsistent in a different way — same for $\mathsf{May}\,\phi$ and $\neg\mathsf{May}\,\phi$
  - If S contains preferences $\mathsf{Must}\,\phi$ would add, $\neg\mathsf{Must}\,\phi$ will remove them
  - If S doesn't contain any of the preferences $\neg\mathsf{Must}\,\phi$ would remove, $\mathsf{Must}\,\phi$ will add them back
- They are dynamically inconsistent: no single perspective can incorporate both simultaneously

Inconsistency, Expressivism and Negation
How Connected to Free Choice?

The Key Link
- To fully solve the negation problem, one needs an expressive account of negation
  - One where negation operates on preferences, rather than propositions
- Precisely that account of negation resolves the tension between Free Choice and Dual Prohibition

- When modals aren't involved connectives behave exactly like classical ones!

Inconsistency, Expressivism and Negation

How Connected to Free Choice?

Varieties of Expressivism

1. Global vs. Local Expressivism
   - Caveat about non-modal language, and other kinds of modality
2. Psychological vs. Social
   - Do deontic modals motivate because they activate preferences?
   - Or because agents are responsive to each other's commitments?
   - Room for a hybrid answer...

Thanks!

(Slides available at `http://williamstarr.net/research`)

Connective Semantics

In Full Detail

Connective Semantics

1. $S[\mathsf{p}] = \{\{w \in s \mid w(\mathsf{p}) = 1\}^{\succsim} \mid s^{\succsim} \in S\}$
2. $S[\neg\phi] = \{s^{\phi^-(\succsim)} - \bigcup(\{s^{\succsim}\}[\phi]) \mid s^{\succsim} \in S\}$
   - $\phi^-(\succsim) := \langle \succ - \{\langle w, w'\rangle \in \succsim_i \mid \{W^{\langle\varnothing,=\rangle}\}[\phi] = \{s_0^{\succsim_1}, \ldots, s_n^{\succsim_m}\} \;\&\; 1 \leqslant i \leqslant m\}, \sim\rangle$
   - $\phi^-(\succsim)$ removes from $\succ$ any pairs that updating with ϕ would add to an empty ordering. For non-expressive discourse this will idle. If $\phi = \mathsf{Must}(\psi)$ this will extract preferences for ψ-worlds over $\neg\psi$-worlds.
3. $S[\phi \wedge \psi] = S[\phi][\psi]$
4. $S[\phi \vee \psi] = S[\phi] \cup S[\phi]$

Deontic Semantics for *Must*

In Full Detail

$$S[\mathsf{Must}(\phi)] = \begin{cases} \{s^{\phi^+(\succsim)} \mid s^{\succsim} \in S\} & \text{if } \forall s^{\succsim} \in S\colon \mathit{Choice}(s^{\phi^+(\succsim)}) = s_\phi^{\succsim} \\ \{\varnothing^{\phi^+(\succsim)} \mid s^{\succsim} \in S\} & \text{otherwise} \end{cases}$$

- $s_\phi^{\succsim} := \bigcup(\{s^{\succsim}\}[\phi])$
  - $s_\phi^{\succsim}$ is the set of ϕ-worlds in s
- $\phi^+(\succsim) := \langle\{\langle w, w'\rangle \in s \times s \mid w \succ w' \text{ or}, w \in s_\phi^{\succsim} \;\&\; w' \in s_{\neg\phi}^{\succsim}\}, \sim\rangle$
  - $\phi^+(\succsim)$ adds to $\succ$ a preference for each $w \in s_\phi^{\succsim}$ over each $w' \in s_{\neg\phi}^{\succsim}$.

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